**T.C.**
**ATATÜRK ÜNİVERSİTESİ**

BİLİMSEL ARAŞTIRMA PROJELERİ KOORDİNASYON BİRİMİ

**PROJE BAŞLIĞI**

Teorik ve Uygulamalı Matematik Bilimleri Sempozyumu

**Proje No**: FBS-2021-9965

Kongre, Sempozyum Düzenleme Projesi

# SONUÇ RAPORU


**Proje Yürütücüsü:**
Doç. Dr. Furkan YILDIRIM
Narman Meslek Yüksek Okulu
Finans-Bankacılık ve Sigortacılık Bölümü

Araştırmacılar
Prof. Dr. Nejmi CENGİZ
Prof. Dr. Tamer UĞUR
Prof. Dr. Kürşat AKBULUT
Prof. Dr. Alper ÇİLTAŞ
Prof. Dr. Alper Cihan KONYALIOĞLU
Doç. Dr. Çağrı KARAMAN
Doç. Dr. Yeşim SARAÇ
Dr. Öğr. Üyesi Fatma SAĞSÖZ
Dr. Öğr. Üyesi Mesut KARABACAK

Dr. Öğr. Üyesi Sait TAŞ
Dr. Öğr. Üyesi Sıdıka Şule ŞENER KILIÇ
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Dr. Öğr. Üyesi Nazlı KARACA
Dr. Öğr. Üyesi Şennur BAKIRTAŞ
Öğr. Gör. Yakup ÇİFTÇİ
Öğr. Gör. Yasin SANCAR
Öğr. Gör. Süha GÖKALP
Arş. Gör. Tuğçe KUNDURACI
Arş. Gör. Merve AKTAY
Arş. Gör. Yeter ÜNLÜ
İdari Personel Emir Sultan GÖKKAYA

Mayıs 2022

ERZURUM

**1. TEMEL BİLGİLER:** Tüm bilgi alanlarını eksiksiz doldurunuz.

| | |
|---|---|
| Kongre/Sempozyum Adı | I. Uluslararası Temel ve Uygulamalı Matematik Bilimlerindeki Güncel Gelişmeler Sempozyumu |
| Kongre/Sempozyum Kısa Adı | ISCDFAMS-2022 |
| Web Sitesi Adresi | https://iscdfams.com/ |
| Kaçıncı Kez Düzenlendiği | 1 |
| Tarihi | Başlama Tarihi: 25.05.2022 Bitiş Tarihi: 27.05.2022 |
| Yeri | Atatürk Üniversitesi Narman Meslek Yüksekokulu |
| Türü | ☐ Ulusal ☒ Uluslararası ☒ Uluslararası Katılımlı |
| Düzenleyen Kurum/Birim | Atatürk Üniversitesi Narman Meslek Yüksekokulu |
| Destekleyen Kurum(lar) | Atatürk Üniversitesi |
| Etkinlik Dili/Dilleri | ☒ Türkçe ☒ İngilizce ☒ Diğer: Tüm Diller |
| Kongre/Sempozyum Başkanı/ Başkanları | ☒ Üniversitemizden<br>☒ Yurt İçindeki Diğer Üniversitelerden<br>☐ Yurt Dışından T.C. Uyruklu<br>☐ Yurt Dışından Yabancı Uyruklu |
| Bilim Kurulu Üye Sayısı | Üniversitemizden: 19<br>Yurt İçindeki Diğer Üniversitelerden: 21<br>Yurt Dışından T.C. Uyruklu: 0<br>Yurt Dışından Yabancı Uyruklu: 68 |
| Kabul Edilen Bildiri Sayısı | Sözlü Sunum: 133 Poster: 0 Toplam: 133 |
| Katılımcı Sayısı | Yurt İçinden: 59 Yurt Dışından: 71 Toplam: 130 |
| Davetli Konuşmacı Sayısı | Yurt İçinden: 3 Yurt Dışından: 6 Toplam: 14 |
| Davetli / Özel Oturum | ☒ Düzenlendi ☐ Düzenlenmedi |
| Sosyal, Kültürel Program | ☐ Düzenlendi ☒ Düzenlenmedi |
| Bildiri Kitabı / Özetler Kitabı | 1. ☒ Bildiri Kitabı ☒ Özetler Kitabı ☐ Yayımlanmadı<br>2. ☐ Basılı ☐ CD/DVD Ortamında ☒ İnternet Üzerinden |
| Bildirilerin Dergide Yayımlanması | 1. ☒ Yayımlanmayacak ☐ Yayımlanacak ☐ Yayımlandı<br>2. ☐ ISI İndekslerindeki Dergi ☐ Alan İndekslerindeki Dergi<br>☐ Diğer Uluslararası Dergi ☐ Diğer Ulusal Dergi<br>3. Yayımlandı İse Mahiyeti ☐ Tüm Bildiriler ☐ Seçilen Bildiriler |
| Bilim Kurulu Değerlendirmesi İle Verilen Ödüller | 1. ☐ Verildi ☒ Verilmedi<br>2. ☐ Bildiri Ödülü Sayısı: ☐ Sunum Ödülü Sayısı:<br>☐ Poster Ödülü Sayısı: ☐ Diğer Ödül Sayısı: |

**2. ETKİNLİĞİN TANITIMI:** Toplantının amacı, kapsamı, kaçıncı kez düzenleneceği, nerede yapılacağı ve beklenen faydalar gibi tanıtım bilgileri özetlenmelidir.

Atatürk Üniversitesi'nin desteğiyle ilk kez düzenlenen 1. Uluslararası Temel ve Uygulamalı Matematik Bilimlerindeki Güncel Gelişmeler Sempozyumu, 23 Mayıs - 25 Mayıs 2022 tarihleri arasında Atatürk Üniversitesi Narman Meslek Yüksekokulu ev sahipliğinde COVID-19 nedeniyle çevrimiçi olarak yapılacaktır. Dünyanın her köşesinden matematikçileri buluşturan bu sempozyumun programı, açılış konuşması, çağrılı ana konuşmalar, dizi konuşmalar ve genç akademisyen konuşmaları ile kısa söyleşilerden oluşmaktadır.

Uzmanlık alanı itibariyle deneyim sahibi olan davetli konuşmacılar ile başarılı akademisyenler teorik ve uygulamalı matematik konularındaki birikimlerini tüm bilim insanlarına aktarma imkanı bulmuş olacaktır. Bu sempozyumda lisans, lisansüstü öğrencileri ve bilim insanları bilgi transferi yapacakları gibi ortak çalışma yürütme şansı elde edeceklerdir. Tüm akademisyenlerin çevrimiçi olarak düzenlenen bu sempozyum ile pandemi koşullarında bir araya gelme şansı yakalayacakları gibi birlikte bilimsel çalışma yapmaya da teşvik edici bir ortam oluşturulacaktır.

**Anahtar Kelimeler:** Analiz ve Fonksiyonlar Teorisi, Uygulamalı Matematik, Diferansiyel Geometri, Geometri, Cebir ve Sayılar Teorisi, Topoloji, Olasılık ve İstatistik, Matematiğin Temelleri ve Matematik Lojik, Bilgisayar Bilimleri.

**3. ETKİNLİĞİN KONULARI:** Toplantı duyurusunda ilan edilen/edilecek konular listelenmelidir.

Aşağıda belirtilen alanlarda tüm katılımcılara açık birer sunum yapılacaktır.

* Teorik Matematik (Küme Teorisi, Matematiksel Mantık, Model Teorisi)

* Cebir ( Sıra Teorisi, Genel Cebirsel Sistemler, Cisim Teorisi ve Polinomlar, Değişmeli Halkalar)

* Sayılar Teorisi (Temel Sayı Teorisi, Analitik Sayı Teorisi, Cebirsel Sayı Teorisi, Aritmetik)

* Geometri (Konveks Geometri, Ayrık ve Kombinatoryal Geometri, Diferensiyel Geometri, Cebirsel Geometri, Aritmetik Geometri, Diyofant Geometri)

* Topoloji (Genel Topoloji, Cebirsel Topoloji, Diferansiyel Topoloji)

* Uygulamalı Matematik ( Olasılık ve İstatistik, Sayısal Analiz, Bilgisayar Cebri, Klasik Mekanik, Parçacık Mekaniği, Sayısal Akışkanlar Dinamiği ve Isı Transferi, Diferansiyel Operatörlerin Spektral Analizi, Diferansiyel Denklemlerde Kontrol, Yöneylem Araştırması, Matematiksel Modelleme, Matematiksel Programlama)

**44. ÖZEL OTURUMLAR ve DAVETLİ KONUŞMACILAR:** Toplantıda özel oturumlar düzenlenip düzenlenmeyeceği ve düzenlenecek ise hangi konularda olacağı belirtilmelidir. Ayrıca Toplantı davetli konuşmacıların konuşmalarına yer verilip verilmeyeceği belirtilmeli, var ise davetli konuşmacıların kimler olduğu ve hangi konularda konuşma yapacakları belirtilmelidir.

Aşağıda isimleri ve üniversiteleri belirtilen davetli konuşmacılar, son yıllarda yaptıkları ve/veya literatürde yapılan diğer akademik çalışmalar/yayınlar/projeler hakkında tüm katılımcılara açık sunum yapacaklardır.

Açılış Konuşması:

1. Prof. Dr. Ahmet IŞIK, Kırıkkale Üniversitesi, Türkiye
*Mathematics and Mathematics Education*

2. Prof. Dr. Arif Salimov, Bakü Üniversitesi-Azerbaycan
*New Developments in the Theory of Lifts*

3. Prof. Dr. Bismark Singh, Friedrich-Alexander Üniversitesi, Almanya
*Optimization Models for Pandemic Response Planning*

4. Prof. Dr. Bayram Şahin, Ege Üniversitesi-Türkiye
*Conformal Riemannian Maps from Kaehler Manifolds to Riemannian Manifolds*

5. Prof. Dr. Josef Mikesh, Palacky University-Çek Cumhuriyeti
*Geodesics Mappings and Their Generalizations*

6. Prof. Dr. Boaz Tsaban, Bar Ilan University-İsrail
*Selection Principles in Mathematics*

7. Prof. Dr. Ljubisa D.R. Kocinac, University of Nis-Sırbistan
*More on Set Versions of Star Selection Principles*

8. Prof. Dr. James Peters, University of Manitoba-Kanada
*Good Covers for Nerve Cell Complexes*

9. Doç. Dr. Murat KİRİŞÇİ, Istanbul Üniversitesi - CerrahpaşaTürkiye
*On the artificial intelligence, big data, blockchain technologies in medicine*

**5. DÜZENLEME EKİBİ:** Toplantıda genel başkan, başkan, eşbaşkan gibi sorumlu görevliler ve organizasyon komisyonu üyeleri, teknik program üyeleri gibi görevlilere ait bilgiler verilmelidir.

| Adı Soyadı | Unvanı | Kurumu | Organizasyondaki Görevi |
|---|---|---|---|
| Ömer Çomaklı | Prof. Dr. | Atatürk Üniversitesi | Kongre Onursal Başkanı |
| Abdulhalik KARABULUT | Prof. Dr. | Ağrı İbrahim Çeçen Üniversitesi | Kongre Onursal Başkanı |
| Bülent ÇAKMAK | Prof. Dr. | Erzurum Teknik Üniversitesi | Kongre Onursal Başkanı |
| Furkan Yıldırım | Doç. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Başkanı |

| | | | |
|---|---|---|---|
| Çağrı Karaman | Doç. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Başkan Yardımcısı |
| Yeşim Saraç | Doç. Dr. | Atatürk Üniversitesi | Genel Sekreter |
| İbrahim Karahan | Doç. Dr. | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Ortak Başkanı |
| Abdullah Çağman | Dr. Öğr. Üyesi | Ağrı İbrahim Çeçen Üniversitesi | Düzenleme Kurulu Ortak Başkanı |
| Nejmi CENGİZ | Prof. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Kürşat AKBULUT | Prof. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Tamer UĞUR | Prof. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Alper ÇİLTAŞ | Prof. Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Ceren Sultan Elmalı | Prof. Dr. | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Üyesi |
| Mustafa Bayram | Prof. Dr. | İstanbul Gelişim Üniversitesi | Düzenleme Kurulu Üyesi |
| Engin ÖZKAN | Prof. Dr. | Erzincan Binali Yıldırım Üniversitesi | Düzenleme Kurulu Üyesi |
| Murat Çağlar | Doç. Dr. | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Üyesi |
| İbrahim Karahan | Doç. Dr. | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Üyesi |
| Mahmut Akyiğit | Doç. Dr. | Sakarya Üniversitesi | Düzenleme Kurulu Üyesi |
| Seher ASLANCI | Doç. Dr. | Alanya Alaaddin Keykubat Üniversitesi | Düzenleme Kurulu Üyesi |
| Fatma KARAKUŞ | Doç. Dr. | Sinop Üniversitesi | Düzenleme Kurulu Üyesi |
| Ali ÇAKMAK | Doç. Dr. | Bitlis Eren Üniversitesi | Düzenleme Kurulu Üyesi |
| Sait Taş | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Mesut Karabacak | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Sıdıka Şule Şener Kılıç | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Nazlı Karaca | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Semra Yurttançıkmaz | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Fatma Sağsöz | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Şennur BAKIRTAŞ | Dr. Öğr. Üyesi | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Süha GÖKALP | Öğr. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Kadirhan Polat | Dr. Öğr. Üyesi | Ağrı İbrahim Çeçen Üniversitesi | Düzenleme Kurulu Üyesi |
| Sibel Turanlı | Dr. Öğr. Üyesi | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Üyesi |
| Muhammed Yiğider | Dr. Öğr. Üyesi | Erzurum Teknik Üniversitesi | Düzenleme Kurulu Üyesi |
| Ayşenur UÇAR | Dr. Öğr. Üyesi | Doğuş Üniversitesi | Düzenleme Kurulu Üyesi |

| | | | |
|---|---|---|---|
| Tuğçe KUNDURACI | Arş. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Merve AKTAY | Arş. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Yeter ÜNLÜ | Arş. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Olgun DURMAZ | Dr. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Zekai AYIK | Dr. | Harran Üniversitesi | Düzenleme Kurulu Üyesi |
| Süha GÖKALP | Öğr. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Yakup ÇİFTÇİ | Öğr. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Yasin SANCAR | Dr. Öğr. Gör. | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |
| Emir Sultan GÖKKAYA | İdari Personel | Atatürk Üniversitesi | Düzenleme Kurulu Üyesi |

Satırlar gerektiği kadar artırılabilir.

**6. ÖNEMLİ TARİHLER:** Toplantı duyurusunda ilan edilen/edilecek önemli tarihleri belirtiniz.

| **Başlık** ([4]) | **Tarih** |
|---|---|
| Toplantı İlan / Kayıt Başlangıç Tarihi | 27 Aralık 2021 |
| Kayıt Başvurusu İçin Son Tarih | 30 Mart 2022 |
| Değerlendirme Sonuçlarının Duyurulması | 15 Nisan 2022 |
| Sempozyum Programı İlan Tarihi | 4 Mayıs 2022 |

([4]) Başlık alanında verilen ifadeler değiştirilebilir, satırlar gerektiği kadar artırılabilir.

**7. ETKİNLİĞİN BÜTÇE ÖZETİ:** Toplantının bütçe özetini beklenen gelirler ve giderleri göz önüne alarak genel hatları ile belirtiniz. Öngörülen gelirler belirlenirken destekleyen kuruluşların sağlayacağı katkılar ve katılımcılardan sağlanacak gelirler vb. tüm hususları dikkate alınız.

| **GELİRLER** | |
|---|---|
| Tanımı/Adı | Tutar |
| 1- Atatürk Üniversitesi Bap Desteği | |
| | |
| **GELİRLER TOPLAMI** | 15.210,01 TL |
| **GİDERLER** | |
| Tanımı/Adı | Tutar |
| 1. Web Sitesi Tasarımı (1 adet x 2.500,00 TL) | 2.500,00 TL |
| 2. Sempozyum Kitabı (1 adet x 3,389,84 TL) | 3,389,84 TL |
| 3. Organizasyon Bedeli (1 adet x 7.000,00 TL) | 7.000,00 TL |
| **TOPLAM** | 12.889,84 TL |
| **KDV (%18)** | 2.320,17 TL |
| **GİDERLER TOPLAMI (KDV DAHİL)** | 15.210,01 TL |

**8. BÜTÇE KALEMLERİ GEREKÇESİ:** BAP Koordinasyon Biriminden talep edilen parasal desteğin her bir kalemi için gerekçe verilmelidir. Benzer nitelikte olan düşük bedelli kırtasiye gibi ortak kullanım amacına sahip tüketim malzemeleri gruplanarak ortak gerekçelendirilebilir.

**1. Web Sitesi Tasarımı:** Sempozyumun daha fazla kesime ulaşması için (Web site kurulumu, Ana sayfa ve slider düzenlenmesi, Web site sunucuya kurulup çalıştırılması).
**2. Sempozyum Kitabı:** E-Kitap İnceleme, Tashih ve Grafiker hizmetleri
**3. Organizasyon Bedeli:** Sempozyumun oluşturulması ve yürütülmesi esnasında kullanılacak tüm ihtiyaçlar (Teknik Destek + Video ve Fotoğraf Çekimi + Grafik Tasarım Hizmeti + Organizasyon Sekreteryası (Hizmeti) + Dijital Katılım Belgesi + Rehberlik + Açılış Organizasyonu + Duyuru Hizmeti + Resmi Yazışmalar + Kayıtların Kurgulanıp Teslim Edilmesi) belirtilmiştir.

**9. TARTIŞMA ve SONUÇLAR:** Projenin öneri aşamasında ortaya konulan hedeflere ne ölçüde ulaşıldığı açıkça ortaya konulmalı, ulaşılamayan hedefler var ise bunların gerekçeleri de tartışılmalıdır. Organizasyon başarımı, etkinliğe gönderilen bildiri sayısı, kabul edilen bildiri sayısı, katılımcı sayısı ve niteliği gibi hususları da göz önünde bulundurunuz.

Atatürk Üniversitesi'nin desteğiyle ilk kez düzenlenen 1. Uluslararası Temel ve Uygulamalı Matematik Bilimlerindeki Güncel Gelişmeler Sempozyumu, 23 Mayıs - 25 Mayıs 2022 tarihleri arasında Atatürk Üniversitesi Narman Meslek Yüksekokulu ev sahipliğinde COVID-19 nedeniyle çevrimiçi olarak yapılmıştır.

Bu sonuç raporunda ise ara rapordan sonra yapılan çalışmalar belgeleri ile sunulacaktır. Sempozyumumuz yukarıda belirtildiği gibi 40'ı ülkemizden 68'i yurt dışından oluşan toplam 128 kişilik güçlü bir Bilimsel Kurulla çalışmaktadır. Projenin kabulünden itibaren, öncelikle sempozyumumuz web sitesi hazırlanmış ve dünyanın çeşitli yerlerindeki matematikçilere ulaşılmıştır. Bunun sonucunda ise başvuru süresi boyunca sempozyumumuza toplam **197** başvuru olmuştur. Bu başvurulardan ise özenli hakem değerlendirmesinden sonra **133**'ü sempozyum için uygun bulunmuştur. 133 başvurunun **124**'ü sunumunu başarılı bir şekilde gerçekleştirmiştir. Sempozyum bildiri kitabı ise https://iscdfams.com/ adresinde https://ekitap.atauni.edu.tr/index.php/1st-international-symposium-on-current-developments-in-fundamental-and-applied-mathematics-sciences-iscdfams-2022-abstract-and-full-text-symposium-book/ bağlantısında ve bu belgenin sonunda yer almaktadır.

# 1st INTERNATIONAL SYMPOSIUM ON CURRENT DEVELOPMENTS IN FUNDAMENTAL AND APPLIED MATHEMATICS SCIENCES

## (ISCDFAMS 2022)

## ABSTRACT AND FULL TEXT SYMPOSIUM BOOK

Editors:
Assoc. Prof. Dr. Furkan YILDIRIM
Asst. Prof. Dr. Fatma SAĞSÖZ

ATATÜRK ÜNİVERSİTESİ YAYINLARI
ATATURK UNIVERSITY PUBLICATIONS

**Eser Adı:**
1st INTERNATIONAL SYMPOSIUM ON CURRENT DEVELOPMENTS IN FUNDAMENTAL AND APPLIED MATHEMATICS SCIENCES (ISCDFAMS 2022) ABSTRACT AND FULL TEXT SYMPOSIUM BOOK

**Editors:**
Assoc. Prof. Dr. Furkan YILDIRIM
Asst. Prof. Dr. Fatma SAĞSÖZ

**Yayınlar Yönetmeni:**
Doç. Dr. Bünyamin AYDEMİR

**Yayın Kurulu:** Prof. Dr. Ahmet SARI, Prof. Dr. Ali UTKU, Prof. Dr. Bülent ÇAVUŞOĞLU, Prof. Dr. Erdinç ŞIKTAR, Prof. Dr. Hakan Hadi KADIOĞLU, Doç. Dr. Bünyamin AYDEMİR, Doç. Dr. Hasan Tahsin SÜMBÜLLÜ

**Dizgi ve Tasarım:** Abubekir KALE

**Atatürk Üniversitesi Yayınları No:** 1298

**ISBN:** 978-625-7086-54-7

**Erişim Adresi:** https://ekitap.atauni.edu.tr

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## CONTENTS

**PRESENTATION**

Dear Participants,

We are proud and happy to host you at the 1st International Symposium on Current Developments in Basic and Applied Mathematics Sciences, with online access facility held at Atatürk University in Erzurum.

Mathematics has a vital role in every aspect of life, especially in science and technology. Mathematics is an integral part of attempts to have a better understanding of the world and ourselves.

Considering the place of mathematics in our lives, it is essential to follow the developments in this field closely in order to keep up with the changes in a constantly developing and changing world.

For example, the concepts of innovation and sustainability in today's world of science enable parallelism with the development of both theoretical and applied fields of mathematics.

In this context, the aim of our symposium is to create a platform where scientists from different fields from all over the world can share their experiences and to provide a lively discussion of the most up-to-date topics in the related fields of mathematics at advanced levels.

In ISCDFAMS 2022, which we organized for the first time, a total of 141 oral presentations, 8 of which will be by invited speakers, will be presented in 8 sessions in 3 parallel halls on different branches of mathematics for 3 days.

We would like to express our most sincere thanks and gratitude to Prof. Dr. Ömer ÇOMAKLI, the rector of Atatürk University, Prof. Dr. Bülent ÇAKMAK, the rector of Erzurum Technical University and Prof. Dr. Abdulhalik KARABULUT, the rector of Ağrı İbrahim Çeçen University, for their support.

Correspondingly, we would also like to thank Scientific Research Projects Coordinatorship, Corporate Communications Directorate, Computer Science Research and Application Center Directorate and Publishing House Directorate of Atatürk University.

Finally, we would like to thank the members of the organizing committee and scientific committee who contributed to the coordination of this event, the invited speakers and all the invaluable scientists who participated with their papers.

We wish our symposium to be beneficial and productive, and we offer our sincere respects.

**Assoc. Prof. Dr. Furkan YILDIRIM**

**Symposium Chair**

## TAKDİM

Saygıdeğer Katılımcılar,

Erzurum'da Atatürk Üniversitesinde düzenlenen I. Uluslararası Temel ve Uygulamalı Matematik Bilimlerinde Güncel Gelişmeler Sempozyumunda sizleri online erişimle ağırlamaktan gurur ve mutluluk duyuyoruz.

Matematik hayatın hemen hemen her alanında, özellikle bilim ve teknoloji alanında hayati bir role sahiptir. Matematik, dünyayı ve kendimizi anlama girişimlerinin ayrılmaz bir parçasıdır. Matematiğin hayatımızdaki yeri dikkate alındığında, sürekli gelişen ve değişen bir dünyada değişimlere ayak uydurabilmek için bu alandaki gelişmeleri yakından takip etmek şarttır.

Örneğin, günümüz bilim dünyasında inovasyon ve sürdürülebilirlik kavramları, matematiğin hem teorik hem de uygulamalı alanlarının gelişimi ile paralellik göstermektedir.

Bu bağlamda sempozyumumuzun amacı, dünyanın her yerinden farklı alanlardan bilim insanlarının deneyimlerini paylaşabilecekleri bir platform oluşturmak ve matematiğin ilgili alanlarındaki en güncel konuların ileri seviyelerde canlı bir şekilde tartışılmasını sağlamaktır.

İlkini düzenlediğimiz ISCDFAMS 2022' de 3 gün boyunca matematiğin farklı dallarını içeren konularda, 3 paralel salonda toplam 8 oturumda 8'i davetli konuşmacılar tarafından olmak üzere toplam 141 sözlü bildiri sunulacaktır.

Sempozyumumuzun düzenlenmesinde desteklerinden dolayı Atatürk Üniversitesi Rektörü Sayın Prof. Dr. Ömer ÇOMAKLI' ya, Erzurum Teknik Üniversitesi Rektörü Sayın Prof. Dr. Bülent ÇAKMAK' a ve Ağrı İbrahim Çeçen Üniversitesi Rektörü Sayın Prof. Dr. Abdulhalik KARABULUT' a teşekkür ve şükranlarımızı sunarız.

Ayrıca Atatürk Üniversitesi Bilimsel Araştırma Projeleri Koordinatörlüğüne, üniversitemiz Kurumsal İletişim Direktörlüğüne, üniversitemiz Bilgisayar Bilimleri Araştırma ve Uygulama Merkezi Müdürlüğüne ve üniversitemiz Yayınevi Müdürlüğüne teşekkürlerimizi sunarız.

Son olarak bu etkinliğin gerçekleşmesinde emeği geçen düzenleme kurulu ve bilim kurulu üyelerine, davetli konuşmacılara ve bildirileriyle katılım sağlayan tüm değerli bilim insanlarına teşekkür ederiz.

Sempozyumumuzun faydalı ve verimli geçmesi dileğiyle en içten saygılarımızı sunarız.

**Doç. Dr. Furkan YILDIRIM**

**Sempozyum Başkanı**

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# KEYNOTE SPEECH

# MATHEMATICS AND MATHEMATICS EDUCATION

(Symposium Opening Speech)

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Erzurum has been a fortress for the Turkish nation throughout history. It is the impenetrable border of the east, the confidant city of trust, hope and success. Erzurum is a city that no foreign county managed to subjugate, like the Palandöken mountain which stands even under difficult conditions. Erzurum is a city that shows the dignity of TURKs to the world with its resistance to the occupation forces in the national liberation struggle. As known, Erzurum is a city where history was rewritten on 23 July 1919. It also locates a university which makes the city an attraction for university education since 1956s.

Dear Rectors,

Dear Deans,

Dear Instructor Scientists,

Dear Young Participant academics and

Dear Press Members

As an academic who is proud to be from Erzurum and Atatürk University, I would like to thank you for honoring the "1st International Symposium on Current Developments in Fundamental and Applied Mathematics Sciences" (ISCDFAMS-2022), organized for the first time with the support of Atatürk University.

First of all, I would like to thank the General Chair of the Congress Assoc. Prof. Dr. Furkan YILDIRIM, and all the organising committee members who contributed to the symposium and I hppe such events will continue.

To begin with, I feel the need to limit the relationship between the title of my talk, "Mathematics and Mathematics Education", and what I will say. Both the concepts of Mathematics and Mathematics Education have a structure that will deeply affect the education of thousands of young people and therefore the future of the country. I know that it is impossible to solve the problems brought up by this structural phenomenon with a symposium opening speech.

In addition, since a part of this speech is not the result of scientific research or data, and it is based on scientific life and experience, it is clear that we will not be able to put forward some problems that may arise as a result of scientific investigations, with a scientific aspect, and we willt beun able to propose scientific solutions. Therefore, I should underline that my statements in general are my personal views and they are not scientific.

As of 1998, the concepts of Mathematics - Mathematics Education and Mathematician - Mathematics Educator in our country have caused vicious conflicts.How come?

In the first studies on the restructuring of the Faculties of Education, a research was carried out such as giving the functions of the Faculties of Science, Letters and Arts and Sciences and Education faculties to their owners. Was this the right approach, I leave it up to you to decide. Even going further, it was decided that MAJOR courses in Education faculties should be given from Major Faculties, considering that the faculty members in Education Faculties had a doctorate degree in the relevant major which is education, ignoring that they were associate professors in the same as Field faculties. As it will be remembered, during the 3,5+1,5-year education period, the courses of the first 3.5 years of education faculties were given by the field faculties that took a while, and with this decision, the discussions of Mathematician and Mathematics Educator created a crisis in the academic field. There have been such crises in the past. For example, pre- crisis mathematics and post- crisis mathematics discussions. At that time, the controversy was so heated that Diderot asked Euler to provide a mathematical proof of the existence of God.

Dear participants;

It is also worth noting that, although the curriculum of the newly opened Foundation, Private or State universities is determined according to today's conditions, the courses of Mathematics and Mathematics Education departments or mathematics and other basic science departments are few in their curricula. It is seen that there is a tendency to social areas persistently.

Whereas:

(K, Cevat, in his 1996 study titled Mathematics and Economics). He states that the difference in development between the USA and European countries is due to the fact that European countries turned to factories and the United States of America turned to basic sciences after the Second World War.

It should not be overlooked that there are three fields of science that have not lost their influence and necessity in the world of science:

1. Medical sciences, 2. Basic sciences, 3. Educational sciences.

Because these areas have been and will stand as long as human being exists. The basic science fields are mathematics, physics, chemistry and biology. From this, it is understood that we need basic sciences in order to sustain our current life better, to have a better understanding of what is happening around , and to better perceive the universe we live on.

From this perspective, as just touched upon, it is a fact that the basic building blocks in the construction of the future of the world are basic science and engineering sciences, which are the practice of science, especially applied mathematics and accordingly computer technology.

Because knowing and doing mathematics is a privilege.

The main thing is to produce new information by structuring this information.

A simple example:

Just as someone who can calculate the area of a parallelogram region and the circumference of a circle can infer the area of the circle himself.

While questioning ourselves and nature, especially by using the language of mathematics push the limits of our minds.

Tymoczko, one of the philosophers of the last years, explained the nature of mathematics in a study he made.

Even the existence of mathematics alone points to the important limits that the human mind can reach. (Tymoczko, 1998).

Again, while discussing what mathematics is in European countries in the 1950s and in our country in the 1990s, mathematics and technology, mathematics and art:

Jerry P. King, the author of the book The Art of Mathematics (TÜBİTAK publications, 1992), asked his wife to emphasize the art he saw at a moment when he was looking directly at his wife's face with love while eating on a night when the moonlight was perfect.

"You're more beautiful than any woman I've ever seen"

says. At that moment, when his wife turned and looked at him, thankfully my wife did not think like a mathematician,

What would he have thought?

He would say my compliment was nonsense, untrue. Being more beautiful than all other women he had seen, he had to be more beautiful than himself, which was impossible. In other words, if there is an element in a set, it is unique. Or, it was unthinkable that a wrong situation such as accepting the divisibility of a number other than zero would be divisible by zero.

Now, Let's try to make sense of mathematics and mathematics education, which does not have a single definition.

Mathematics is the science of space and quantity, at the same time, mathematics is a language of reality and an art in itself. Mathematics is a communication skill. Or, it is the common name of sciences that examine the properties of quantities on the basis of numbers and measures, such as arithmetic, algebra, geometry.

Since mathematics is a science based on numbers and measurement, it examines abstract entities and the relations between them. These reviews are based on reasoning.

Mathematics, if viewed correctly, has not only truth, but also extraordinary beauty. It is such a beauty that it is cold, intolerant and harsh, like a statue. He does not care for any part of our weaker nature, he does not care for the magnificent ornaments of paintings and music. Yet he is extraordinarily pure and gifted with a ruthless perfection that only the greatest art can display (Bertrand Russell).

Mathematics education is a system of teaching and learning of mathematics based on a method, technique or a strategy.

Or

It is an effort to understand the nature of mathematics. It can also be considered as an effort to teach what and how.

In the historical process of humanity, we, as mathematicians, have tried and are trying to develop new techniques and methods in teaching mathematics, as well as establishing new theorems in order to evaluate the Field and Field Education together, as in the concepts of Education-Teaching. These techniques may vary according to the skill of the teacher and the learner. For instructors who are authorised to teach concepts, each area of expertise may need a different teaching method in its own branch.

Considering the concepts of mathematics and mathematics education, which do not have a single definition, let's briefly review the conceivable situations.

For a philosopher, the question why mathematics is attractive may come to mind, as well as the question why the language of the universe was written with mathematics (Shapiro, 2000).

Why the practice of abstract results of mathematics to concrete concepts is so obvious.

Despite being so abstract, how does mathematics not fall into a vacuum?

Are mathematical truths absolute truths?

As if he knew about Shapiro's testimony and other judgments,

The famous physicist Oppenheimer, one of the architects of the atomic bomb, said at a conference that today's philosophers do not know mathematics, and even one step further, that their mathematicians do not know mathematics. It is not clear in Oppenheimer's statement whether he meant that mathematicians lacked technical knowledge or simply did not mean that mathematicians did not really understand the essence of what they were dealing with. Here, in my opinion, the probability of emphasizing the technical side is low.

Because 20th century mathematics has been the golden age of mathematics. So what Oppenheimer's means is the nature and nature of mathematics.

Actually, the question what mathematics is, is a very difficult question in the philosophy of mathematics.

Indeed, if you ask a physicist what physics is or a historian what history is, it will not be difficult to answer. However, if you ask a mathematician what mathematics is, the relevant person d can rightly say that he does not know the answer, and this answer cannot prevent him from being a mathematician (Barrow, 2001; 1).

The most important feature that distinguishes mathematics from other sciences is that it is a completely human product. In other words, if there were no human beings, there would be physics, chemistry, biology, geology and astronomy, but there would be no such thing as mathematics (Kart, 1996)

Not knowing what mathematics is should not mean not knowing mathematics.

But I am an academic who does not believe that someone who does not know mathematics can and will do mathematics education, which we tested in various exams.

ESPECIALLY; Since the associate professor exams are exams in which the scientific field is signed, academics with insufficient infrastructure in the relevant discipline should be much more careful. Just because the associate professor application score is more than 100 points, I can't understand why those who have nothing to do with the field have some expectations from the related field, such as applying to an associate professor position from an unrelated branch.

Because; What I will say now should be considered in the academic field. We can gather the reasons why teaching and learning mathematics is difficult under the following four items.

1. The absence of fairy tales in mathematics
2. It is a problem to use mathematical intelligence at any time (Kart, C. 1996)
3. The instructor has not sufficiently assimilated the concepts to be taught
4. The learner has not sufficiently assimilated the concepts he thinks he has learned (IŞIK, A., 2002)

Unless we bring up the negative situations we live in, we will not be able to reach the real truths.

Example of teaching the multiplication of two numbers to third grade primary school students.

I have always wondered about teaching mathematics to a prospective classroom teacher who stated that the sound produced when two eggs hit each other is the multiplication of two numbers.

Or

217

+29

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507

I present you the mathematical knowledge and skills of someone who finds the sum of the numbers 507 and claims that the result is correct. It is really sad that someone with or without this information has high expectations from different fields.

In every quarter century we live in, the importance of education and knowledge is increasing rapidly. From this point of view, it becomes clear why mathematics education and learning mathematics, which are directly related to technology and the quality of living standards, are important. Because without mathematics education, it is not easy to talk about development, economy, science and technological progress in a country. The applicability of learning mathematics, acquired mathematical knowledge and mathematical thinking should be embodied if possible and explained with examples without creating mathematics anxiety (Işık, A., Çiltaş, A. & Bekdemir, M., 2010).

It is enough to look at the environment a little carefully to see the mathematical system in every area of our lives. It is not possible to be a member of a developed democratic society, either today or tomorrow, without mathematical literacy, based on the idea that mathematics, which emerged as a result of the collaboration of mathematicians and electronics, has become more and more perfect (Hardy, G. 999). Because many civilizations, from industry to technology, are indebted to mathematics (Işık, A. & Bekdemir, M, 1998).

In this case, wouldn't the following question come to mind from 7 to 70? How does the learner learn whether he has math anxiety or not, how does learning take place in the brain.

As mathematicians and mathematics educators, one of the most difficult questions that keep our minds busy is; how does learning mathematical concepts happen in the brain?

Just as logic is the youth of mathematics and the adult version of logic in mathematics;

Mathematics education is also the youth of mathematics and the adult form of mathematics education in mathematics.

Permanent learning is possible with successful teaching.

Keywords: Mathematics, Mathematics Education

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# INVITED SPEAKERS

# Optimization Models for Pandemic Response Planning

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Optimizing the spatiotemporal allocation and distribution of a limited number of critical medical resources — such as vaccines and antivirals — is a pervasive problem that is especially aggravated during a pandemic. The challenge is further complicated by mismatches between supply and demand over time and also by uncertainty in demand and/or supply. We present a collection of optimization models that we have developed over the past ten years, and applied to the 2009 H1N1 (retrospectively) and current COVID-19 (dynamically) pandemics. Some of these questions, that lie at the interface of epidemiology and optimization, include how to fairly allocate vaccines, how to determine an optimal set of facilities to provide antivirals, and how to relax and reinstate lockdowns.

Keywords: pandemics, optimization, epidemiology.
2020 Mathematics Subject Classification: 90C15, 90C06, 90C11.

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# NEW DEVELOPMENTS IN THE THEORY OF LIFTS

(Invited Speaker)

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*(Dedicated to the memory of Professor Kentaro Yano)*

The main purpose of this report is to study the differential geometrical objects on tangent bundle corresponding to holomorphic objects of holomorphic manifold. As a result of this algebraic approach we find a new class of lifts (deformed complete lifts) in the tangent bundles.

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# Geometric Realization of Path Cycles as Flow Cycles. Extension of J.H.C. Whitehead Homotopy System Geometric Realization Theorem

James F. Peters[1]

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This talk introduces geometric realization of path cycles in a cell complex as flow cycles in a vector field. A planar cell complex [7-8] is a collection of 0-,1- and 2-cells that may or may notbe attached to each other in a Hausdorff space [2]. A 0-cell is point (either by itself or on a curveor in the intersection between curves in a vector space). A 1-cell is edge (arc), which is an arcattached to a pair of 0-cells. A path $h$ is a continuous map $h : [0, 1] \to X$ with endpoints $h(0)$ , $h(1) \in X$ and all $h(t) \in X$ for $t \in [0, 1]$ (unit interval). From [2], every path has a geometric realization as an edge (1-cell). A path triangle is a sequence of three overlapping paths with no end path. From [2], we also have that every path triangle has a geometric realization as a 2-cell (triangle). In its simplest form, a free group presentation of path triangle is a Rotman free group [6], geometrically realized as a collection of path-connected vertexes [3]. The focus here is on path cycles geometrically realized either as 1-cyles or as a flow cycle in the Euclidean plane. A flow cycle is a collection as path-connected vectors in a vector field with no end vector. A cycle vector field is a flow cycle in a vector field. An optical flow is an example of a cycle vector field. A 1-cycle is a collection of path-connected vertexes (also vectors in a circulation) with no end vertex. From [4], every path cycle is realizable as a 1-cycle with many applications [6] (inspired by S. Lefschetz [1]) such as optical flow fields in video frame sequences.

Keywords: Cell, Cell Complex, Circulation, Flow Cycle, Path, Path Cycle, Vector Field..
2020 Mathematics Subject Classification: 05C38 (Paths and cycles), 57M20 (Low Dimensional Complexes), 37C27 (Periodic orbits of vector fields and flows).

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# Conformal Riemannian Maps from Kaehler Manifolds to Riemannian Manifolds

(Invited Speaker)

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**Abstract**

In the manifold theory, the theory of smooth maps between smooth manifolds is very important. The main reason of the importance of this concept is that this theory has applications in physics (general relativity) as well as in engineering ( computer aided design). One can see in differential geometry, popular smooth maps, in the aspect of the most active area, are isometric immersions and Riemannian submersions One can see that isometric immersions and Riemannian submersions are very special maps comparing with maps of arbitrary ranks. Therefore, Fischer defined Riemannian maps between Riemannian manifolds as a generalization of isometric immersions and Riemannian submersions. A Riemannian map is a map such that it satisfies partially isometry condition. This enables us to consider general isometry between Riemannian manifolds. The purpose of this talk is to present recent developments on conformal Riemannian maps from Kaehler manifolds. Geometric notions in almost Hermitian manifolds are defined according to the complex structure on the manifolds. Considering this approach, invariant, anti-invariant, semi-invariant and slant classes of conformal Riemann maps are introduced, examples are given for each class, characterizations are obtained and the properties of the map are presented. In addition, results are given on whether the map of these classes is totally geodesic and harmonic. Considering that these presented conformal Riemann maps are a broad class of almost Hermitian manifolds including holomorphic submersion, anti-invariant Riemannian submersion, semi-invariant submersion and slant submersion, these new geometric concepts seem to have the potential to stimulus new research problems in differential geometry.

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# Selection Principles in Mathematics

Boaz Tsaban[1]

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I will provide an overview of the theory of Selection Principles, with its connections and applications to various branches of mathematics, including measure theory, function spaces, Ramsey theory, and game theory. I will also talk about *omission of intervals*, a method that I developed for constructing subsets of the real line with extraordinary combinatorial properties, and answering some classic questions.

Keywords:Selection Principles, Omission of Intervals, strong measure zero, Frechet-Urysohn spaces, Menger space, Ramsey theory, game theory.

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# On the Artificial Intelligence, Big Data, Blockhain Technologies in Medicine

(Invited Speaker)

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What is the Blockchain?

To answer this question, we first need to remind some fundamental concepts.

These concepts are Data, Database, Network Technologies, cryptology, and the philosophy of this new idea.

**Data**

As it is known, this word is the same in English and Latin languages, and it is a concept that has started to normalize in our language in daily life. It is the name given to the crude, raw piece of information.

Data is obtained by measurement, counting, experiment, observation, or research.

Like any symbolic representation, data is a set of abstract statements about a particular object, individual, or phenomenon.

Not only in scientific or technological studies but also daily life, we know that data alone has no meaning and function. What makes data meaningful is that, after they are collected, they are grouped, ranked and summarized, processed, and transformed into information. Thus, data gains the power to explain the element they belong to and they become able to serve a purpose such as problem-solving or decision making.

Contrary to popular belief, data is not only produced by humans. Millions of different data such as cosmic rays originating from solar flares that we cannot record and electrons revolving around atoms are constantly produced in the universe. What is more interesting is that the genetic coding that forms the basis of life, namely DNA, is among the most basic data records that have not been handed by humanity, can be copied and reproduced between cells, and are reconstructed by transferring them between the transmitter and the receiver under certain conditions.

From the discovery of writing to today's technologies, there have always been data records. These data have been recorded in bye opportunities given by the history and the conditions of the day.

Recently, while recordings are kept by various physical tools from high-capacity DVDs to USBs, today's indispensable recording source is cloud services.

**Database**

The structure created by librarianship and archiving techniques, whose origins date back hundreds of years, with computer systems is called a database.

Edgar Codd, working at the IBM San Jose Research Laboratory in 1973, introduced the definition of "Relational Database". Relational databases store data in tables and create links between these tables. Users are concerned not with how data is stored, but with how it is presented to them. The language created for querying recorded data is SQL, and this query language has turned into a database query standard.

The first regular data storage solutions were called Spreadsheet Applications.

Considering that spreadsheet applications are limited data storage and analysis solutions, today's huge databases can contain trillions of rows of data and their size can be at the level of Petabytes.

In today's world, cloud solutions provide us with the infrastructures we need with a unique cost advantage. However, a cloud solution is not also the endpoint of technological development. There is more.

Today, the number of devices connected to the Internet has exceeded the human population. While this structure, called the Internet of Things (IoT), grows exponentially, the economy it creates is considered to be trillions of dollars. Is it possible to record and store the data produced by all these systems in a distributed way?

The answer to this question has been YES at the turn of the century. This answer appeared as a kind of data storage solution called "Peer To Peer - P2P" developed for sharing data over machines.

In these systems, the data is not in a single center but distributed or shared over millions of machines. Some of these machines may contain the entire data, while others may contain a partial piece of data.

However, these services, which are thought to be accessed without paying any price, have a price, even if it is not known. While you pull the data into your computer, you also serve as a data source to other users in return for the data you receive.

Therefore, although the platforms that provide these services offer a solution beyond the cloud, they are not a secure storage solution for corporate or private personal data, as the content is not encrypted and does not offer options as to where the data will be stored.

**Network Technologies**

Thanks to technology, periods of one hundred years pass faster. Although we do more in less time! The fact that wireless communication technologies, which play an important role in communication today, actually constitute one of the cornerstones of the natural ecosystem for millions of years, maybe one of the most interesting pieces of information.

Almost all plants have been using wireless communication technology for millions of years to continue their generation. Flowers have pollen and other plants have nuclei for wireless transmission of DNA, one of the oldest data records.

Here, we call the technologies inspired by nature network technologies.

Mobile Web access is no longer a luxury since wireless network technologies started to develop and WiFi standards came into our lives. Today, communication networks that reach very high speeds are one of the most important components that enable the rapid spread of Blockchain technology.

**Cryptology**

Let's talk briefly about the concept of Cryptology, which is the basis of blockchain technology and has great importance, and let's get into our subject.

For thousands of years, one of the most important needs that emerged while recording the produced data has been to hide the data from unwanted eyes. This naturally brings up cryptology, which is a science of thousands of years, starting from the idea of secrecy.

In its simplest form, Cryptography refers to the encryption of data.

Encryption turns any dataset into a seemingly random dataset using a rule structure. This seemingly random dataset can only be converted back to its original and meaningful form by those who have the key used for encryption. For those who don't have this key, it doesn't make any sense. Thus, no matter where or how encrypted data is stored, it remains meaningful only to the owner of the key.

**Philosophy**

It is possible to explain the philosophy of blockchain technology as follows:

The technical study titled "Bitcoin: Peer-to-Peer Electronic Cash Payment System" published by the pseudonymous Satoshi Nakamoto, which emerged only two months after the bankruptcy of Lehman Brothers in 2008 and whose identity is still confidential, although a series of it presents the mathematical and technological application to us the main message is given in the article is as follows: "O people! Whether you know each other or not, it is now possible to establish a secure data recording system without the need for centralized structures. Because this system uses the possibilities of mathematics and technology, it cannot be manipulated or corrupted."

**What is the Blockchain?**

While talking about Blockchain, it is necessary to understand what is in this system, that is, what changes this business rather than the technical analysis of the business. So it is very we must understand chess in our lives.

So, first, we had a single copy of a recording, then we distributed that recording to several computers, then we distributed many copies of this recording to many computers, and finally, each computer has become to keep a record of the transaction. The main reason for this is that the costs have decreased drastically over time.

The development process in digital technologies is so fast that costs decrease inversely compared to technological progress every few years.

It has been seen that it is practically possible to distribute data to many computers over cheap communication networks. At this point, our records have been and continue to be copied to all systems.

This approach is called the Distributed Ledger.

Simply put, Blockchain is a recording technology. So what you call blockchain is a registry.

It is a ledger mentioned here and this ledger is not the only one. This ledger is a common ledger and copies of this ledger are available in many parts of the world.

We heard about Blockchain with Bitcoin. So some misconceptions arose. There is only one ledger on Blockchain and there is a fundamental misconception that this ledger is called bitcoin. No, it's not like that. It is possible to talk about many ledgers and only one of these ledgers is Bitcoin. Since Bitcoin is the first and is very well known, this is mistaken. There are many ledgers besides Bitcoin: For example, Ethereum, avalanche, and Solana. The purpose of recording each of these ledgers is different. In other words, the purpose of existence of these notebooks is different.

Shortly, we will be talking about the health ledgers used by all states. But it should be well known that even though it is expressed as the solution to all problems, the blockchain is not a magic wand either.

**Networks**

For the implementation of the blockchain, it will be sufficient to look at three main parameters:

Having more than one stakeholder.

Stakeholders are generating data.

Having a problem of trust between stakeholders

Yes, let's be careful here. We are not talking about stakeholders trusting each other. On the contrary, blockchain can be implemented in a situation where there are stakeholders who do not trust each other. In other words, we are talking about a structure that can be used by stakeholders who do not trust each other.

**Implementation**

We can think of blockchain applications for the health sector as follows.

1. Personal Health Data
2. Smart Contracts
3. IoT

**1. Personal Health Data**

There is an important concept that enters our lives with Blockchain: Personal sovereignty. This concept comes from here. As a person, my data belongs to me and no one else has the right to use this data without my permission. In other words, no one else can use my data without my permission and no one else can produce value from this data.

Let's exemplify this situation with an economic field. For example, we can earn 20 dollars from the use of our simplest data (such as name-surname, age, gender, place of birth), and 50 dollars from the use of slightly more qualified data (graduate degree, habits, friends, hobbies, etc.). In other words, if our data is on the blockchain and this data is used with our permission, it is possible to earn a good income. Likewise, health data is our private data and should be used as long as we allow it.

The "permission" we use here is "the private key", "the public key" in the blockchain world. In other words, if a person or an institution wants to access and use my data thanks to my key, they have to get my permission. For example, the pharmaceutical industry can use my data in drug production studies. Naturally, when I permit to use these keys, these usages should have a return for me. The blockchain world makes this possible. We said that the person can also earn from this data usage. There are companies in the world that do this type of work. One of them has created a health token, and when a person allows their own health data to be used, they can earn a health token in return for this use permission. This is an interesting and beautiful application.

**2. Smart Contracts**

Here, insurance companies can benefit from blockchain to offer solutions that make life easier, reduce financial losses and prevent fraud.

Smart contracts are essentially a piece of code. So it is software. This software implements predefined rules.

The bond of trust and "promise to pay" inherent in the insurance industry is important. Traditional certificates have been withheld for years by both public and private institutions to create evidence to the client. The software type certificates used during these processes bring great costs to the users. Certain major companies providing these certificates are overpaid. These certificates are tied to a central service provider in a specific chain of rules. Although it may seem like a personal benefit, it can actually be controlled from one point. For example, for humans, this could be a birth certificate or a document such as a driver's license. For consumers, it is in the form of similar certificates that provide verification of quality and authenticity. Blockchain technologies can provide us with systems that will eliminate this system and will be used between stakeholders in real terms and can be used without being tied to a central point.

Blockchains allow these traditional certificates to be stored at an immutable date, where anyone can search and apply. It can update these records as new events occur. It enables the creation of a new type of identity for both people and other objects. These processes are built on a traditional model in which a certificate authority issues certificates.

**3. IoT**

IoT devices on the blockchain network can now autonomously broadcast their records and update the current state they are in. In addition, these devices can speak for themselves, publishing and sharing their histories and identities with third parties. Therefore, it allows the human element to gradually disappear.

In this sense, IoT can be a life-enhancing solution for pharmaceutical supply based on logistics.

In the health sector, recording IoT data on the blockchain is a suitable solution, especially in the cold chain (such as vaccines) or in matters where the supply needs to be tracked. Therefore, it is possible to determine that the transportation is done correctly and that things are going well, without the need for any end-to-end trust.

There is a vaccination card application related to the pandemic, which is one of the most important issues of the recent period, and vaccination, which is one of the forms of protection. This card, which appeared during the coronavirus pandemic, is actually a record and therefore a data. When you save this record on a blockchain, when you are asked to document your vaccination status, you can easily handle your transactions by showing your vaccination information on the blockchain, regardless of any authority. This blockchain-based registration system allows you to transact with verified data without relying on personal trust. Thus, a world emerges where speed, convenience, and costs are reduced. The idea of having the vaccination card on the blockchain is a very good practice.

**Opportunities and Challenges**

How will the digitization in the health sector be or how does it happen? With the spread of the Internet, such questions and expectations came to the fore. Because there is a problem: A patient never only goes to a hospital, the patient doesn't just see a physician. In other words, a patient goes to different physicians and different hospitals. Therefore, it was said that it is very important to share the data related to the diseases that the person has accumulated over the years, with different physicians, and to different hospital systems. However, the infrastructure for this could not be established. It was said that whether there are large buildings, or large servers should be established in these buildings, everyone and all hospitals and health institutions should automatically send patient data to this center, and it should be shared from this center.

Although this idea seemed very good at first, serious problems arose. First of all, a very expensive system was mentioned. There were also security problems. Then the problem of which data to send appeared. On the other hand, one hospital kept data on the same subject in one format with a particular patient information system, while another hospital kept data in a different format. This simply posed the following problem: Where do we send this data and in what format do we keep it? Moreover, the question of what would happen if these data were stolen came to the fore. In other words, it was said that if the data about a person's illness or deficiency, for example, fell into the hands of an insurance company or other types of institutions or persons that would profit from it, it would be misused, and everyone stood up for the data against these problems. In other words, this project is over before it even started, and hospitals and health institutions said that this data belongs to us and we will not share it with anyone. Besides, what will happen if the real owner of the data says that I am paying for this health service and I must move my data to where I want? So things got messed up!

So, has this problem been solved today? In other words, how should this data be kept, and in what format has it been fully established? Unfortunately, the answer will be NO.

Although everyone believes in the necessity of sharing health data all over the world today, the problem is getting more complicated. Because it is not very clear who should share this data and under what conditions. The issues of "who will keep this data and where" have not been clarified. Why am I saying this? Namely, a person's data from an imaging device, ECG data, laboratory data, physical therapy measurements, patient diagnosis data, and treatment data take up a huge amount of space. Are there fields that can hold this much data? At this point, the blockchain is capable of producing solutions to these problems we have expressed.

Blockchain technically solves the problem of storing data and puts the data under the control of the person in terms of protecting the data of the person. Thus, the person can create immutable records and share this immutable data. But sharing is still a problematic issue and even though its technical infrastructure has been formed, legal base problems continue. Many institutions in the world do not put forward commercial concerns and say that we do not share data. Statements like "data are too valuable" or "data is too expensive" are due to misunderstanding. Let's even say this, who will value a piece of data and how?

One of the problems is that states have not yet created legal situations suitable for blockchains. Here, it can be thought that the states take these things slowly and act a bit clumsy. However, for example, if it is not clear in which format the data will be shared, what kind of legal mechanism will be created for this data sharing? Even physicians, not hospitals, or even civil servants in healthcare institutions handle data in different ways. Naturally, the question of how data in different formats will be shared and with whom this data will be shared are unanswered questions. In other words, it is said that all data should be shared with the state regulation, but for example, do Hospital A exchange information with Hospital B? If states give such an order, it will only add to the uncertainty.

World health expenditure is over 8 trillion dollars. This is a huge market. Damage from data breaches in this market is said to be around $100 billion. When we follow the movements of such a large market, we see that without waiting for the regulations, especially the elders of the health sector started these works and I think it seems that the regulations can follow these studies.

Keywords: Data, Database, Cryptology, Network Technologies, Philosophy.

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# Geodesic mappings and their generalizations

(Invited Speaker)

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The important part of the contemporary differential is a study of mappings that map geodesics onto special curves, i.e., geodesics, almost geodesics, analytic curves, etc. The theories of these mappings have applications in theoretical physics for modeling gravitational, electromagnetic, and other fields.

Geodesic mappings are mappings that map any geodesic onto geodesic. Let $A_n = (M, \nabla)$ and $\bar{A}_n = (\bar{M}, \bar{\nabla})$ be affine spaces, where $M$ and $\bar{M}$ are $n$-dimensional manifolds, $\nabla$ and $\bar{\nabla}$ are torsion-free affine connections. It is known [4] that $A_n$ admits geodesic mapping onto $\bar{A}_n$ if and only if the *Levi-Civita equations*

$$\bar{\nabla}_X Y = \nabla_X Y + \psi(X)Y + \psi(Y)X \tag{1}$$

hold for any tangent fields $X, Y$ and where $\psi$ is a linear differential form, $\nabla$ and $\bar{\nabla}$ are the Levi-Civita connections. We can suppose that $\bar{M} = M$. If $\psi$ is vanishing then the mapping $f$ is *affine* or *trivially geodesic*.

It was proved that geodesic mapping of geodesic mappings of (pseudo-)Riemannian spaces onto (pseudo-)Riemannian spaces preserve the differentiability class of metric [2]. The similar holds for geodesic mappings of spaces with affine connection onto (pseudo-)Riemannian spaces [3].

Furthermore, it was also proved that all affine spaces are geodesically equivalent to equiaffine spaces [10]. Mikeš and Berezovski [5] found a criterion (equivalent to the Levi-Civita equations) for the existence of geodesic mapping of equiaffine space onto (pseudo-)Riemannian space. The criterion was in a form of linear partial differential equations in covariant derivatives. These equations hold globally and also locally. See [4].

A special part is also dedicated to the existence of so-called geodesic bifurcation. This term describes the situation where there exist more geodesics passing through the given point in the given direction, the case where the condition of geodesic uniqueness is not fulfilled. The example of geodesic bifurcation was given on the surface of revolution. Moreover, based on this example, another example of closed geodesic was given as well as an example of geodesic bifurcation of product spaces [9].

Next, S.G. Leiko defined so-called rotary mapping. Rotary mappings are mappings that map any geodesic onto an isoperimetric extremal of rotation.

Isoperimetric extremal of rotation is a special curve which is an extremal of functional of length and functional of rotation. These curves also have an interpretation in physics - they are trajectories of particles with a spin. We generalized this term and found a counterexample of spaces that admit rotary mappings [6].

Finally, we mention the basics of the projective transformation and holomorphically projective transformations (of K¨ahler manifolds) theory. In the end, we also briefly mention the basics of $F$-planar mappings, their definitions, and certain results of these theories [1, 4, 7, 8].

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# More on set versions of star selection principles
(Invited Speaker)

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Star selection principles have been introduced in 1999 by Kocinac [1]. In this talk we present some results on the recently introduced and studied set star covering properties (Kocinac, Konca, Singh [2,3,4,5]). A space X is said to have the set star-Menger property if for each nonempty subset A of X and each sequence $(U_n : n \in \mathbb{N}\ )$ of collections of open sets in X such that $\overline{A} \subset \cup U_n$ , $n \in \mathbb{N}$, there is a sequence $(V_n : n \in \mathbb{N}\ )$ such that for each $n \in \mathbb{N}$, $V_n$ is a nite subset of $U_n$ and $A \subset \cup_{n \in \mathbb{N}} St(\cup V_n, U_n)$ We also discuss set strongly star-Menger, set (strongly) star Hurewicz, and set selectively ccc spaces. Some lines of further investigation are considered.

Keywords: Set star Menger, set strongly star Menger, set star Hurewicz, set selectively star ccc.

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# ABSTRACTS

# An Existence Study for a Tripled System with $p-$Laplacian Involving $\varphi-$Caputo Derivatives

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In this work, we present the existence and uniqueness of solutions for a tripled system of fractional differential equations with nonlocal integro multi point boundary conditions by using the $p-$Laplacian operator and the $\varphi-$Caputo derivatives. The presented results are obtained by the two fixed point theorems of Banach and Krasnoselskii. The following problem:

$$\begin{cases} \mathcal{D}_{0^+}^{r_{1m};\varphi}\psi_p\left[\mathcal{D}_{0^+}^{r_{2m};\varphi}\left(u_m(t)-\mathcal{I}_{0^+}^{\sigma;\varphi}G_m(t,u_1(t),u_2(t),u_3(t))\right)\right]=H_m(t,u_1(t),u_2(t),u_3(t)), \\ m=\overline{1,3},\ and\ t\in J=[0,1] \\ \psi_p\left[\mathcal{D}_{0^+}^{r_2;\varphi}\left(u_m(t)-\mathcal{I}_{0^+}^{\sigma;\varphi}G_m(t,u_2(t),u_3(t))\right)\right]\big|_{t=0}=0, \\ u_m(0)=0,\ u_m(1)=i=13\sum\lambda_{im}u_i\left(\zeta_{im}\right),\zeta_{im}\in[0,1] \\ \varphi\left(1\right)-\varphi(0)=K>0. \end{cases} \tag{1}$$

Here, we take $\mathcal{D}_{0^+}^{r_{im};\varphi}, i,m=\overline{1,3}$ as the $\varphi-$Caputo fractional derivatives of orders $r_{im}$, $0\leq r_{1m}<1<r_{2m}<2$, and $\mathcal{I}_{0^+}^{\sigma;\varphi}, 0<\sigma$, the fractional integral of order $\sigma$, $\lambda_{im}\in\mathbb{R}_+^*$, and $\varphi: J\to\mathbb{R}$ is an increasing function such that $\varphi'(t)\neq 0$, and $\psi_p(z)=z\left|z\right|^{p-2}$ denotes the $p-$Laplacian operator and satisfies $\frac{1}{p}+\frac{1}{q}=1, (\psi_p)^{-1}=\psi_q(q\geq 2)$. For all $t\in J$, $G_m, H_m: J\times\mathbb{R}^3\to\mathbb{R}$ is a given functions satisfying some assumptions that will be specified later.

Keywords:$p-$Laplacian operator, $\varphi-$Caputo derivative, existence of solution, fixed point.
2020 Mathematics Subject Classification: 30C45; 39B72; 39B85.

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# Analysis and applications of the proportional Caputo derivative and integral

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In this paper, we study the Langevin equation within the generalized proportional fractional derivative. The proposed equation involves a variable coefficient and subjects to mixed integrodifferential boundary conditions. We introduce the generalized proportional fractional derivative and expose some of its features. We mainly investigate the existence, uniqueness and different types of Ulam stability of the solutions via fixed point theorems and inequality techniques. Finally, we provide an example to support our main results.

Keywords: Fractional Langevin equation; Generalized proportional fractional derivative; Fixed point theorem.
2020 Mathematics Subject Classification: 26A33, 34A08, 34B15.

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# On a solvable system of rational difference equations of higher order

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In this paper, we present that the following system of difference equations

$$x_n = \frac{x_{n-k} z_{n-l}}{b_n x_{n-k} + a_n z_{n-k-l}},\ y_n = \frac{y_{n-k} x_{n-l}}{d_n y_{n-k} + c_n x_{n-k-l}},\ z_n = \frac{z_{n-k} y_{n-l}}{f_n z_{n-k} + e_n y_{n-k-l}},$$

where $n \in \mathbb{N}_0$, $k, l \in \mathbb{N}$, the initial values $x_{-i}, y_{-i}, z_{-i}$ are non-zero real numbers for $i \in \overline{1, k+l}$, and sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(c_n)_{n\in\mathbb{N}_0}$, $(d_n)_{n\in\mathbb{N}_0}$, $(e_n)_{n\in\mathbb{N}_0}$ and $(f_n)_{n\in\mathbb{N}_0}$ are real, can be solved in closed form. We describe the forbidden set of the initial values using the obtained formulas and also determine the asymptotic behavior of solutions for the case $k = 3$, $l = 1$ and the sequences $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(c_n)_{n\in\mathbb{N}_0}$, $(d_n)_{n\in\mathbb{N}_0}$, $(e_n)_{n\in\mathbb{N}_0}$ and $(f_n)_{n\in\mathbb{N}_0}$ are constant. Our results considerably extend and improve some recent results in the literature.

Keywords: System of difference equations, Closed solution, Forbidden set.
2020 Mathematics Subject Classification: 39A10, 39A20, 39A23.

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# On Eight Solvable Systems of Difference Equations in Terms of Generalized Padovan Sequences

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In this study we show that the systems of difference equations

$$x_{n+1} = f^{-1}\big(af(p_{n-1}) + bf(q_{n-2})\big), \quad y_{n+1} = f^{-1}\big(af(r_{n-1}) + bf(s_{n-2})\big),$$

for $n \in \mathbb{N}_0$, where the sequences $p_n$, $q_n$, $r_n$ and $s_n$ are some of the sequences $x_n$ and $y_n$, $f : D_f \longrightarrow \mathbb{R}$ is a "$1-1$" continuous function on its domain $D_f \subseteq \mathbb{R}$, initial values $x_{-j}$, $y_{-j}$, $j \in \{0, 1, 2\}$, are arbitrary real numbers in $D_f$ and the parameters $a, b$ are arbitrary complex numbers, with $b \neq 0$, can be explicitly solved in terms of generalized Padovan sequences. Some analytical examples are given to demonstrate the theoretical results.

Keywords: System of difference equations, solution of explicit form, Padovan number.
2020 Mathematics Subject Classification: 39A10, 39A20.

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# On the boundary observability and controllability of the wave equation in some non-cylindrical domains

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The goal of this talk is to study the small vibrations of axially moving strings described by a one-dimensional wave equation in a bounded interval with moving endpoints. We establish a sharp energy estimate for the solution. Then, we give explicit observability inequalities in a sharp time at each endpoint. Moreover, by using the Hilbert uniqueness method we obtain exact boundary controllability results.

Keywords: 1-d wave equation, non-cylindrical domain, energy estimates, boundary observability, Hilbert uniqueness method.
2020 Mathematics Subject Classification: 35L05, 93B05, 93B07.

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# THE ATTITUDES AND SELF-EFFICIENCIES OF HIGH SCHOOL STUDENTS CONTINUED THE DISTANCE MATHEMATICS COURSE AGAINST DISTANCE EDUCATION DURING THE COVID-19 PANDEMIC

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The aim of this study is to determine the attitudes and self-efficacy of distance education of high school students who participated in the mathematics course through distance education during the Covid-19 pandemic period. The students of a girls' high school in Erzurum city center participated in the study. The survey method, which is one of the quantitative research designs, was used in the study. Data were collected with scales prepared to determine students' attitudes and self-efficacy. The collected data were analyzed descriptively and the attitudes and self-efficacy of the students towards distance mathematics teaching were presented.

Keywords: Mathematics, Mathematics Education, Distance Edication

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# Existence and uniqueness results for nonlinear hybrid implicit Caputo-Hadamard fractional differential equations

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Text of the abstract:

In this paper, we use the Banach fixed point theorem to obtain the existence, interval of existence and uniqueness of solutions for nonlinear hybrid implicit Caputo-Hadamard fractional differential equations. We also use the generalization of Gronwall's inequality to show the estimate of the solutions.

Keywords: Implicit fractional differential equations, Caputo-Hadamard fractional derivatives, fixed point theorems.

2020 Mathematics Subject Classification: 34A12, 34K20, 45N05.

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# On the asymptotic behaviour of a non-local eigenvalue problem

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In this paper, we consider a non-local eigenvalue problem in some weighted spaces of Sobolev type. Applying the anisotropic singular perturbations method to establish the existence of the principal eigenvalue and its associated eigenfunction. Besides, we obtain some proprieties like the simplicity of this eigenvalue and the positivity of its eigenfunction. Then we describe the asymptotic behaviour of the solution as $\varepsilon \to 0$

Keywords: Anisotropic singular perturbations, eigenvalue problem, non-local problem, eigenvalues and eigenfunctions.

2020 Mathematics Subject Classification: 35B25; 35B40; 45K05; 35J20; 47A75.

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# On the Controllability of Some Systems on Lie Groups

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Linear control systems on matrix Lie groups have an important place in terms of their application to real-life problems [1]. By the definition in the context of Lie groups a linear control system is determined by the pair $\Sigma = (G, \mathcal{D})$, where the state space is a real finite dimensional Lie group $G$ with the Lie algebra $L(G)$ and the dynamic $\mathcal{D}$ is given by the family of differential equations on $G$

$$\dot{g(t)} = X(g(t)) + \Sigma_{i=1}^{n} u_i(t) Y^i(g(t))$$

where $X$ is an element of normalizer of $L(G)$ and called drift vector field, and the control vectors $Y^1, Y^2, \ldots, Y^n$ belong to $L(G)$, where $L(G)$ denotes the Lie algebra of left-invariant vector fields. The control function $u = (u_1, \ldots, u_n)$ is in the class of piece-wise constant functions from $[0, \infty)$ to $\mathbb{R}^n$. The main purpose of geometric control theory is to investigate whether it is possible to reach any other state from a given specific state in a positive time via admissible trajectories. For example, from a given initial condition $x_0$, can a new condition $x_1$ be reached by transferring $x_0$ via the admissible control $u$ in a positive time? Considering this for disease and epidemic models, is it possible to find a medical strategy to transform an initial level of disease, at another final level of health, in a positive time [2]? In this presentation, we work on the controllability properties of some kind of control theory problems.

Keywords: Linear control systems, matrix lie groups, dynamical systems, controllability.
2020 Mathematics Subject Classification: 93B05, 93C05, 22E25.

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# Solving the absolute value equation based on a new smoothing function

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In this paper, we solving the absolute value equation $Ax - |x| = b$, denoted by $AVE$, where $A$ is an arbitrary $n \times n$ real matrix and $b \in \mathbb{R}^n$ by the smoothing-type algorithm. Using a smooth approximation of the absolute function, we reformulate $AVE$ as a system of smooth equations and propose a new smoothing function. We show that the algorithm is well-defined when the singular value of $A$ exceed one and under the same assumption the algorithm is convergent. Finally, we make some comparisons between this new function and some previously defined functions to predict its effectiveness.

Keywords: Absolute value equations, smoothing function, smoothing Newton algorithm.
2020 Mathematics Subject Classification: First, Second, Third.

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# Schauder and Banach fixed point theorem for semilinear fractional problem

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The main objective of this paper is study the existence and uniqueness of distributional solution in a fractional Sobolev space for a semilinear fractional problem that contains a nonlocal operator. Thanks to suitable conditions on the semilinear term we prove the existence result and we use the application of the Schauder fixed-point theorem. Furthermore, By the Banach contaction principle theorem we establish in this paper the uniqueness of distributional solution in particular case. In fact, Fixed-point theorems is particulary useful for proving existence of solutions to nonlinear partial differential equations and it is also applicable in the fractional cases.

Keywords: Partial differential equation, Distributional solution, semilinear fractional equation.
2020 Mathematics Subject Classification: 35J16, 35A16, 31C25.

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# Growth of solutions of linear fractional differential equations with polynomial coefficients

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Consider the linear differential equation

$$f^{(n)} + P_{n-1}(z) f^{(n-1)} + ... + P_1(z) f' + P_0(z) f = 0, \tag{1}$$

where $P_0(z) \not\equiv 0, P_1(z), ..., P_{n-1}(z)$ are polynomials. It is well known that every solution $f$ of equation (1) is an entire function of finite rational order $\sigma(f)$ satisfying

$$\sigma(f) \leq 1 + \max_{0\leq k\leq n-1} \frac{\deg P_k}{n-k};$$

see [2, 4, 7, 8]. In [1], Gundersen et al gave the possible orders of solutions of (1). The question which arises here: what about linear fractional differential equations?

Fractional order differential equations have become a very important tool for modeling phenomena in many diverse fields of science and engineering which traditional differential modeling cannot accomplish.(see, for example, Kilbas et al. [3]). In present, three kinds of fractional derivatives are often used, the Grünwald Letnikov derivative, the Riemann Liouville derivative and the Caputo derivative. There are many discussions for properties of these derivatives, see [5, 6]. All these studies are limited in real line.

This talk is devoted to the study of the growth of solutions of the linear fractional differential equation

$$\frac{r^{q_n}}{z^{[q_n]}} \mathcal{D}^{q_n} f(z) + P_{n-1}(z) \frac{r^{q_{n-1}}}{z^{[q_{n-1}]}} \mathcal{D}^{q_{n-1}} f(z) + ... + P_1(z) \frac{r^{q_1}}{z^{[q_1]}} \mathcal{D}^{q_1} f(z)$$
$$+ P_0(z) f(z) = 0.$$

involving the Caputo fractional derivatives by using the generalized Wiman-Valiron theorem in the fractional calculus. Illustrative examples are given.

Keywords: Linear fractional differential equations, growth of solutions, Caputo fractional derivative operator.
2020 Mathematics Subject Classification: 34M10, 26A33.

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# BLOW UP OF SOLUTION OF A NONLINEAR WAVE EQUATION WITH GENERAL SOURCE AND DAMPING TERMS

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**Abstract:**
In this work, we study a wave equation with general source and damping terms. Under some assumptions on the source and the damping terms, we show that the solution blows up in fnite time.

Keywords: Wave equation, General source term, General damping term, Blow up.

2020 Mathematics Subject Classification: 35L05, 35B40, 35L70, 93D20.

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# Non autonomous iterative differential inclusion

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In this work, we study the existence and uniqueness of a certain type of non autonomous differential inclusion involving a maximal monotone operator (depending on the time) and compositions of the unknown function, it has the following form

$$-\dot{u}(t) \in A(t)u(t) + f(t, u(t), u^{[2]}(t), \cdots, u^{[n]}(t));\ u(0) = u_0, \tag{1}$$

where $A(t) : D(A(t)) \subset \mathbb{R} \rightrightarrows \mathbb{R}$, $[0,T] \subset D(A(t))$ is a maximal monotone operator and $f : [0,T] \times \mathbb{R}^n \to \mathbb{R}$ is a Carathéodory Lipschitz mapping and satisfies the linear growth condition. Then, we present a Bolza-type example for an optimal control problem associated with (1) where the controls are Young measures.

Keywords: Iterative, differential inclusion, Bolza problem.
2020 Mathematics Subject Classification: 34K35, 28B20, 34K35.

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# Growth of solutions of linear fractional differential equations with entire coefficients

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In the study of the growth of solutions of the classical linear differential equation

$$f^{(n)} + A_{n-1}(z) f^{(n-1)} + ... + A_1(z) f' + A_0(z) f = 0, \qquad (1)$$

where the coefficients are entire functions many authors are interested in the following question: what conditions on the coefficients will guarantee that every solution $f(z) \not\equiv 0$ of (1) has infinite order? In the literature, there are many papers concerning this question; see for example [1, 3, 5, 6, 7]. The main tool used is this investigation is the logarithmic derivative estimates, see [4]. Unfortunately, up to now, there is no similar estimates given in [4] for the fractional derivatives except the Wiman-Valiron theorem in the fractional calculus that is valid only on a neighborhood of the points $z$ where the function reaches its maximum, see [2]. Despite this obstruction, we will investigate the growth of solutions of the linear fractional differential equations

$$\frac{r^{q_n}}{z^{[q_n]}} \mathcal{D}^{q_n} f(z) + A_{n-1}(z) \frac{r^{q_{n-1}}}{z^{[q_{n-1}]}} \mathcal{D}^{q_{n-1}} f(z) + ... + A_1(z) \frac{r^{q_1}}{z^{[q_1]}} \mathcal{D}^{q_1} f(z)$$
$$+ z A_0(z) f(z) = 0.$$

involving the Caputo fractional derivatives. Under some conditions we prove that every non trivial solution is of infinite order.

Keywords: Linear fractional differential equations, infinite order of growth of solutions, Caputo fractional derivative operator.
2020 Mathematics Subject Classification: 34M10, 26A33.

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Sufficient conditions for global asymptotic stability of a kind of nonlinear neutral differential equations

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Abstract: This work addresses the stability study for nonlinear neutral differential equations. Thanks to a new technique based on the fixed point theory, we find some new sufficient conditions ensuring the global asymptotic stability of the solution. In this work we extend and improve some related results presented in recent works of literature. An example is exhibited to show the effectiveness and advantage of the results proved.

Keywords: contraction mapping principle; asymptotic stability; neutral differential equation.

2020 Mathematics Subject Classification: 34K20, 34K30, 34K40.

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# Dynamical behavior of a differential-algebraic system with fractional order

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The objective of the present work is to investigate the dynamics of a fractional-order differential-algebraic predator-prey system with Holling type III functional response. This model can be established as follow:

$$\begin{cases} D^{q_1}x &= x\left(r\left(1-\dfrac{x}{K}\right)-\dfrac{axy}{d+x^2}\right), \\ D^{q_2}y &= y\left(s\left(1-\dfrac{y}{N}\right)+\dfrac{bx^2}{d+x^2}-E\right), \\ 0 &= E(py-c)-v, \end{cases} \tag{1}$$

The boundedness and positivity of solutions for this model are derived. Local stability of the ecosystem near the coexistence equilibria have been thoroughly investigated when the economic profit $v$ varies in both commensurate and incommensurate fractional orders. The influence of the commensurate fractional orders on the existence of the Hoph bifurcation for the fractional-order ecosystem is explored. Finally, numerical illustrations are performed in order to validate some of the important analytical findings.

Keywords: Differential-algebraic system, Fractional order, Stability, Hopf bifurcation, Harvesting.
2020 Mathematics Subject Classification: First, Second, Third.

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# Historical-Philosophical Development and Teaching of Mathematical Objects

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The nature of mathematics, mathematical objects and the emergence of these objects are important in terms of history and philosophy (Font, Godino &Gallardo, 2013). Discussion of the emergence and teaching of mathematical objects as a philosophy of mathematics problem has been going on since Plato (Rozov, 1989). Knowing the ontology, epistemology of mathematical objects and where they come from has a role in designing instructional environments. Is the existence of mathematical objects the same as the existence of an object in everyday life? Are the existence of the phone and the development of the concept of function similar?In this context, this research aims to discover the nature of mathematical objects by analyzing their existence forms, historical roles and existence in mathematics teaching. It attempts to explain how mathematical objects emerge from mathematical applications and, ultimately, to provide a functional answer to the fundamental question of how mathematical knowledge is structured in schools.

Keywords: Mathematical objects, teaching mathematics

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# Some Aspects of Interchanging Difference Equation Orders

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Difference equations can be useful in applications which are matrix-oriented and in time located data where the functional behaviour between measurements is not known or may be subject to ill-conditioning [5]. This paper considers some aspects of connections between similar, but different, expressions of the same sequences of numbers. For example, Roettger and Williams have used cubic extensions of the Lucas sequence to develop simple, but not easy, tests for primality [1,2,3], Williams having long been a foremost authority on such tests [4]. This paper builds on Roettger and Williams' extensions of the primordial Lucas sequence to consider some relations among difference equations of different orders. This paper utilises some of their second and third order recurrence relations to provide an excursion through basic second order sequences and related third order recurrence relations with a variety of numerical illustrations which demonstrate that mathematical notation is a tool of thought.

Keywords: Second order recurrence relations, primordial sequence, Vandermonde determinant.

2020 Mathematics Subject Classification: 11B37, 11B39, 11B50.

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# Universal Covering of a Lie Group

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It is well known that any homomorphism between Lie algebras extends to the Lie groups if the domain is simply connected. This result together with the construction of a Lie group structure on the universal covering of a given Lie group provide that we obtain a description of connected Lie groups from simply connected Lie groups. In this talk, we mainly intend to explain how a Lie group structure is obtained on the simply connected universal covering of a given Lie group.

Keywords: Covering space, universal cover, lie groups.
2020 Mathematics Subject Classification: 22A10, 22E20, 57S05.

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# Semi continuous perturbations for nonconvex sweeping process

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Our aim in this work is to prove a general existence result for

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + F(t, x(t)) + G(t, x(t)) \text{ a.e. in } [0, T] \\ x(t) \in C(t), \forall\, t \in [0; T] \\ x(0) = x_0 \in C(0), \end{cases}$$

where $C : [0, T] \rightharpoonup \mathbb{R}^n$ is a set-valued mapping with nonconvex noncompact values, $N_{C(t)}(x(t))$ denotes the Clarke normal cone to $C(t)$ at $x(t)$. $F : [0, T] \times \mathbb{R}^n \rightharpoonup \mathbb{R}^n$ and $G : [0, T] \times \mathbb{R}^n \rightharpoonup \mathbb{R}^n$ are two set-valued mappings with closed nonconvex values, namely $F$ is taken measurable, integrably bounded such that $x \longmapsto F(t, x)$ is lower semicontinuous. $G$ satisfies a linear growth condition with compact values and takes mixed values in the following sense: for every $t \in [0, T]$, at each $x \in \mathbb{R}^n$ such that $G(t, x)$ is convex, $G(t, \cdot)$ is upper semicontinuous, and whenever $G(t, x)$ is nonconvex, $G(t, \cdot)$ is lower semicontinuous on some neighborhood of $x$. The result is based on a set-valued version of the fixed point theorem. In the study of existence of solution for differential inclusion, the use of convexity assumptions is widely acknowledged. This property is not true, in general, when convexity is dropped. The nonconvex case has been studied by various approaches. In [1] and [2], the authors introduced a class of mapping mixing both upper semicontinuous and lower semicontinuous regularity assumptions, by defining a set-valued mapping with decomposables values, which contains a set-valued selection with convex values.

Keywords: Nonconvex differential inclusions, Sweeping process, Normal cone, Prox-regular set, Mixed semicontinuous, Fixed point theorem.
2020 Mathematics Subject Classification: 34A60.

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# Simpson Type Inequalities for Katugampola Fractional Integral

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In mathematics, Katugampola fractional operators are integral operators that generalize the Riemann–Liouville and the Hadamard fractional operators into a unique form.

Let $[u, v] \subset \mathbb{R}$ be a finite interval. Then the left and right-side Katugampola fractional integrals of order $\alpha > 0$ of $\Psi \in X_c^p(u, v)$ are defined by

$$^{\rho}I_{a+}^{\alpha}\Psi(\varepsilon) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_a^{\varepsilon} \frac{\eta^{\rho-1}}{(\varepsilon^{\rho} - \eta^{\rho})^{1-\alpha}} \Psi(\eta)\, d\eta,$$

and

$$^{\rho}I_{b-}^{\alpha}\Psi(\varepsilon) = \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_{\varepsilon}^{v} \frac{\eta^{\rho-1}}{(\eta^{\rho} - \varepsilon^{\rho})^{1-\alpha}} \Psi(\eta)\, d\eta,$$

with $u < \varepsilon < v$ and $\rho > 0$, respectively.

In this paper, we will give some inequalities for Katugampola Fractional integrai inqualities using by Simpson Type inequalities.

Keywords: Simpson inequality, Katugampola fractional integral, harmonic convex.
2020 Mathematics Subject Classification: 26D15, 26D10, 34A08.

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# A note on L and R topologies

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In this work, we obtain some useful results on left operand and right operand topologies produced by a raw binary operation which is weaker than both partial and multivalued binary operations concepts.

Keywords: L topology, R topology, raw binary operation.
2020 Mathematics Subject Classification: 22A15, 22A30, 22A99.

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# Non-convex valued perturbation of first order problems with maximal monotone operators

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**Abstract**

Let $H$ be a finite dimensional Hilbert space and let $I := [0, 1]$ be an interval of $\mathbb{R}$. In this work, we study the existence of absolutely continuous solutions of the following first-order differential inclusion

$$\begin{cases} -\dot{u}(t) \in A(t, u(t))u(t) + F(t, u(t)) & \text{a.e. } t \in I \\ u(0) = u_0, \end{cases}$$

where $A(t, x) : D(A(t, x)) \subset H \rightrightarrows H$ is a time and state-dependent maximal monotone operator and $F : I \times H \rightrightarrows H$ is a non-convex perturbation.

Keywords: Differential inclusion, maximal monotone operator, non-convex perturbation.
2020 Mathematics Subject Classification: 34A60, 34G25, 49J52, 49J53.

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# A Dynamic electroviscoelastic problem with thermal effects

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We consider a mathematical model which describes the dynamic process of contact between a piezoelectric body and an electrically conductive foundation. We model the material's behavior with a nonlinear electro-viscoelastic constitutive law with thermal effects. Contact is described with the Signorini condition, a version of Coulomb's law of dry friction. A variational formulation of the model is derived, and the existence of a unique weak solution is proved. The proofs are based on the classical result of nonlinear first order evolution inequalities, the equations with monotone operators, and the fixed point arguments.

Keywords: Electro-viscoelastic, Fixed point, Coulomb's friction law.
2020 Mathematics Subject Classification: 74M15, 74M10, 74F05, 49J40.

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# On unique solvability and Picard's iterative method for absolute value equations

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In this paper, we deal with unique solvability and numerical solution of absolute value equations (AVE), $Ax - B|x| = b$, $(A, B \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n)$. Under some weaker conditions, a simple proof is given for unique solvability of AVE. Furthermore, we demonstrate with an example that these results are reliable to detect unique solvability of AVE. These results are also extended to unique solvability of standard and horizontal linear complementarity problems. Finally, we suggest a Picard iterative method to compute an approximated solution of some uniquely solvable AVE problems where its globally linear convergence is guaranteed via one of our weaker sufficient condition.

Keywords: Absolute value equations, Linear complementarity problems, Linear system, Singular value, iterative methods.
2020 Mathematics Subject Classification: 65F08, 90C33, 93C05..

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# Limit cycles of a class of planar polynomial differential systems

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**Abstract**

In this paper we study the maximum number of limit cycles that can bifurcate from a linear center, when perturbed inside a class of planar polynomial differential systems of arbitrary degree $n$. Using averaging theory of first and second order, we estimate the maximum number of limit cycles that this class of systems can exhibit.

Keywords: Limit cycles, Averaging theory, Kukles systems, Lienard systems.
2020 Mathematics Subject Classification: 34C29, 34C25, 47H11.

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# A Derivative-Free algorithm for continuous global optimization

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In this presentation, we consider the bound constrained global optimization problem of the following form:

$$f^* = \min_{x \in D} f(x). \tag{1}$$

where $D = \prod_{i=1}^{n} [L_i, U_i] \subset \mathbb{R}^n$ with $L_i, U_i$ are real numbers for $i = 1, \ldots, n$ and the real objective function $f(x)$ is only continuous. The problem (1) is of interest in many real-world problems involving objective functions which are only continuous and do not possess strong mathematical properties (such as convexity, differentiability, Lipschitz or Hölderian continuity etc.). If a function $f$ is a priori known to be only continuous, then apart from saying that $f$ "remains near $f(x)$ in a neighbourhood of $x$" which translates into dealing with the modulus of continuity of $f$, at $x$ nothing tractable can be inferred on the values of $f$ away from $x$. One way to tackle such enormous uncertainties is to use the following result:

**Theorem 1** *[3] Let $f$ be a real function defined on a compact set $D \subset \mathbb{R}^n$. Then $f$ is continuous if and only if for all $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that for all $x, x' \in D$, we have:*

$$|f(x) - f(x')| \leq C_\varepsilon \|x - x'\| + \varepsilon \tag{2}$$

A drawback of the above theorem is that given $f$ and $\varepsilon$, there are no specific means to recover exactly the constant $C_\varepsilon$ for example for black-box functions. Our idea is to work on a sequence $\{C_j\}_{j\in\mathbb{N}}$ of positive constants that controls the growth of $C_\varepsilon$. This is done by actual collecting of information while running a Lissajous curve throughout the feasible domain.

The proposed method is based on the reducing transformation technique by running in the feasible domain a single parametrized Lissajous curve, which becomes increasingly denser and progressively fills the feasible domain. By means of a one-dimensional search algorithm, we realize a mixed method which explores the feasible domain. To speed up the mixed exploration algorithm, we

have incorporated a derivative-free local search algorithm to explore promising regions. This method converges in a finite number of iterations to the global minimum. The simulation results based on a set of 180 benchmark functions with diverse properties and different dimensions show the efficiency and the abilities of the proposed algorithm in finding the global optima compared with the existing methods.

Keywords: Global optimization, Local optimization, Reducing transformation method, Lissajous parametrized curve.

2020 Mathematics Subject Classification: 90C26, 90C90..

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# Diophantine approximation by prime numbers of a special form

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We show that whenever $\delta > 0$, $\eta$ is real and constants $\lambda_i$ subject to certain assumptions, there are infinitely many prime triples $p_1$, $p_2$, $p_3$ satisfying the inequality $|\lambda_1 p_1 + \lambda_2 p_2 + \lambda_3 p_3 + \eta| < (\max p_j)^{-1/18+\delta}$ and such that, for each $i \in \{1, 2, 3\}$, $p_i + 2$ has at most 7 prime factors. The proof uses Davenport - Heilbronn adaption of the circle method together with a vector sieve method.

Keywords: Circle method, almost primes, diophantine inequality.
2020 Mathematics Subject Classification: 11D75, 11N36, 11P32.

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# Cohen positive strongly p-summing m-homogeneous polynomials from a tensor viewpoint

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**Abstract**

We study the concept of Cohen positive strongly p-summing m-homogeneous polynomials and we present a duality relationship of this class in tensor terms.

Keywords: Homogeneous polynomials, Positive $p-$summing operators, Tensor norm.

2020 Mathematics Subject Classification: 46A20, 46A32, 47B10.

# The main result

**Definition** Let $1 < p \leq +\infty$ and $m \in \mathbb{N}$. An $m-$homogeneous polynomial $P : X \longrightarrow F$ is Cohen positive strongly $p-$summing, if there is a constant $C > 0$, such that for any $x_1, ..., x_n \in X$ and any $y_1^*, ..., y_n^* \in F^*$,

$$\sum_{i=1}^{n} |\langle P(x_i), y_i^* \rangle| \leq C \left( \sum_{i=1}^{n} \|x_i\|^{mp} \right)^{\frac{1}{p}} \|(y_i^*)_{i=1}^n\|_{\ell^n_{p^*,|weak|}(F^*)} . \tag{1}$$

The class of such polynomials is denoted by $\mathcal{P}^+_{Coh,p}(^mX; F)$. It is a Banach space with the norm $d_p^+(.)$ which is the smallest constant $C$ such that the inequality (1) holds.

**Proposition (Duality)** Let $P : X \to F$ be an $m-$homogeneous polynomial. The following are equivalent

(i) $P$ is Cohen positive strongly $p$-summing.

(ii) $\phi_P$ is bounded on $\left((\otimes_s^m X) \otimes F^*, \lambda_p^+\right)$.

Under this circumstances $d_p^+(P) = \|\phi_P\|$.

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# Analysis of a Electro-Elastic contact problem with wear and unilateral constraint

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We study a mathematical problem describing the quasistatic frictional contact with wear between a piezoelectric body and a moving foundation was considered. The evolution of the wear function is described with Archad's law. A nonlinear electro-elastic constitutive law is used to modelled with a normal compliance condition with unilateral constraint and the associated with regularized Coulomb's law of dry friction, which takes into account the wear of the foundation. We drive a variational formulation for the model, in the forme of a coupled system for the diplacement, the electric potential and the wear. An existence and uniqueness result was proved. The proofs were carried out by using arguments of elliptic variational inequalities, differential equations, and Banach fixed point theorem.

Keywords: Electro-Elastic; Friction contact, fixed point.

2020 Mathematics Subject Classification: 74C10, 49J40, 74M10

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# Comparison of Two Effective Methods on Numerical Solutions of Differential Equations

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In positive sciences, modeling of processes that change over time often leads us to differential equations. Except for some known special solution procedures an analytical solution of most differential equations cannot be found. For this reason, studies to find approximate solutions of differential equations in different formations have always been interesting. For this purpose, there are many numerical methods in the literature. In this study, the differential transform method [1,2] and the Taylor collocation method [3,4] will be emphasized. Both methods are Taylor series based methods and aim to find the coefficients in the Taylor series expansion of the approximate solution. While the differential transform method transforms the given differential equation into an algebraic equation, the Taylor collocation method transforms the given differential equation into a system of algebraic equations at the ordering points. In obtaining this algebraic equation and algebraic equation system, derivative based transformations of functions and derivatives are used in the differential transform method, while matrix representations of functions and derivatives are used in the Taylor collocation method. With the above mentioned methods using derivative based transformations, faster and easier calculations can be made compared to integral transform methods.

In this study, the comparison of the mentioned two methods on some ordinary differential equations through their offered by on the solutions will be included.

Keywords: Differential transform method, Taylor collocation method, Ordinary differential equations.

2020 Mathematics Subject Classification: 34A25, 65L60, 34B15.

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# A Theoretical Synthesis of Philosophy of Mathematics and Mathematical Beliefs and Application to Mathematics Education

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In the last quarter-century, studies on the philosophy of mathematics and mathematical beliefs have increased. However, researchers have argued that philosophy and mathematics education remain unconnected in classrooms. An answer is important to the following question: How the philosophy of the people and the program and the beliefs of the teachers and the students affect the learning process? Today, there is a need to re-imagine mathematics education and to build it on philosophical fundamentals.

Ernest (1991) defined a teacher's philosophy of mathematics as his/her personal theory of the nature of mathematical knowledge. The term "belief" is often used synonymously with terms such as disposition (e.g., Scheffler, 1965), world view (e.g., Schoenfeld, 1985), perception (e.g., Gopnik & Melzoff, 1997), philosophy (e.g., Ernest, 1991; Lerman, 1983) and so forth. Since these concepts are not directly observable and because of their overlapping nature, they are confused with each other, and even these concepts are often defined in terms of each other. For example, Ernest (1985) and Lerman (1990) emphasized a teacher's application of the philosophy of mathematics as a basis for his/her beliefs about teaching and learning mathematics. In addition, Ernest (1989a, 1989 b ) stated that a teacher's understanding of the nature of mathematics constitutes his or her belief system concerning mathematics as a whole and forms the basis of her philosophy of mathematics.

The system of mathematical beliefs provided a model for characterizing the teacher's personal philosophy of mathematics and its evolution. According to this model, a mathematics teacher can have three main philosophical conceptions: Instrumentalist, Platonist, and Problem-solving (see Ernest, 1989a, 1989b). For example, the Platonist philosophical view claims that mathematical knowledge has "a priori", is immutable and eternally truth. Hence a teacher with this view tries to convey this eternal truth to the students in a better way as a more rigid and finished product in a more or less didactic-receptive way.

It is aimed to express a theoretical synthesis of the philosophy of mathematics and mathematical beliefs, their relations with each other in the present study. These mathematics-oriented teaching and learning beliefs are part of the teacher's larger system that determines how the philosophy of mathematics is applied (Ernest, 1991). There are many ways in which processes theories, concepts, and results of past enquiry in philosophy can be applied to mathematics education (Ernest, 1998; Skovsmose, 1994). Through our call in this presentation, we hope that more researchers will be stimulated to revisit the integration of philosophy into mathematics education, and to conceptualize what, how, where, and when efforts should be focused to integrate the aspects of philosophy.

Keywords: Philosophy of mathematics, Mathematical beliefs, Mathematics education, Therotical synthesis

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# Truncated condition for second order perturbed sweeping process

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The sweeping process is a particular differential inclusion governed by a normal cone to a moving set. This type of problem plays an important role in elasto-plasticity and dynamics. The naming of "sweeping process" is due to the fact that $u(t)$ is swept by $D(t)$. In the present paper, we are mainly interested to study the following new variant of the sweeping process

$$(\mathcal{S})\begin{cases}\ddot{u}(t) \in -N_{D(t,u(t),\dot{u}(t))}(\dot{u}(t)) - G(t,u(t),\dot{u}(t)), & \text{a.e.} \quad t \in [0,T];\\ u(0)=u_0, \dot{u}(0)=v_0 \in D(0,u_0,v_0), \end{cases}$$

where $N_{D(t,u(t),\dot{u}(t))}(\dot{u}(t))$ stands for the Clark normal cone to the closed set $D(t,u(t),\dot{u}(t))$ at a point $\dot{u}(t)$, $D$ is an unbounded and uniformly $r$ prox reguler set depending jointly on time, state and velocity, and $G$ is set valued perturbation forces, that is external forces applied on the system. The authors [1] have studied the same problem under the ball-compactness assumption for $D(t,u(t))$, then various extensions have been obtained by many authors. Recently, in the infinite-dimensional space $H$, we extend the results obtained by [2] by adapting the implicit discretization scheme to the nonconvex case. The standard Lipschitz (or absolutely continuous ) assumption is replaced by a truncated one, in order to deal with a large class of unbounded sets. Also, we weaken the assumptions on the unbounded set-valued perturbation by taking only the element of minimum norm satisfying a linear growth condition.

Keywords: Moreau's sweeping process, prox-regular set, truncation.
2020 Mathematics Subject Classification: 34K09, 49J52.

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# On Quasi Ideals of Nearness Semigroups

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In 1956, the concept of quasi ideals for semigroups [1] and rings was firstly defined by Steinfeld. Many researchers studied important properties for quasi ideals. Readers can find several paper about quasi-ideals in [2, 3, 4].

In 2002, Peters introduced near set theory that is a generalization of rough set theory [5]. In this theory, Peters defined an indiscernibility relation by using the features of the objects to determine the nearness of the objects [6]. Afterwards, he generalized approach theory of the nearness of non-empty sets resembling each other [7, 8]. In 2012, İnan and Öztürk investigated the concept of nearness groups [9] and other algebraic approaches of near sets. Also, Tekin defined quasi ideals in semirings on weak nearness approximaion spaces [10]. The aim of this paper is to study the notion of quasi-ideals in semigroups on weak nearness approximation spaces and explain some of the concepts and definitions.

Keywords: Weak nearness approximation spaces, nearness semigroups, quasi ideals.
2020 Mathematics Subject Classification: 03E75, 03E99, 16D25.

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# BERGE EQUILIBRIUM IN RANDOM BI-MATRIX GAME

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We consider a bi-matrix game where each player has a stochastic matrix payoff. First, We formulate the game with chance-constraints. Then, in the case where the entries of the payoff matrices are normal distributions, we prove the existence of Berge equilibrium. Finally, we show that this equilibrium can be obtained by solving an equivalent quadratic problem.

Keywords: Bi-matrix game, noncooperative game, stochastic matrices,Berge equilibrium.
2020 Mathematics Subject Classification: 91A05, 91A10, 15B51.

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# The Dirichlet Problem for the Polyanalytic Equations in a Ring Domain

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The theory of boundary value problems for complex partial differential equations has important applications in some physical problems. As an example of boundary condition, explicit solutions to Dirichlet problem for some type of complex PDEs are given for different particular domains, such as the unit disc, ring, see e.g. [1, 2]. Furthermore, basic or mixed boundary value problems for higher order complex partial differential equations have been studied in [3, 4, 5, 6, 7].

In this work, we investigate solution of Dirichlet boundary value problem for polyanalytic functions in a concentric ring domain $R = \{z \in \mathbb{C} : 0 < r < |z| < 1\}$ in a explicit form. We used similar techniques as in [1, 2, 8].

Keywords: Dirichlet problem, polyanalytic equation, ring domain.
2020 Mathematics Subject Classification: 30E20, 30E25, 32A55.

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# Resolution a problem of quantum mechanics in fractional dimensional space

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The purpose of this work is to obtain an exact solution of an equation describes the spin-0 particles with relativistic energy subjected to the action of a scalar potential and a vector potential in the context of the fractional-dimensional space, where the momentum and position operators fulfils the R-deformed Heisenberg algebras. Therefore, several problems has been solved, and in all cases, the expressions of the eigenfunctions are determined and expressed in terms of the special functions, and the corresponding eigenvalues are exactly obtained and established depending on the deformation parameters $D$ and $\sigma$, which explains the confinement in law dimension.

Keywords: Fractional-dimensional space, special functions, R-deformed Heisenberg algebras.
2020 Mathematics Subject Classification: 00A69, 34A08, 46N50..

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# Locally I- connectedness

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In this paper, we introduce locally connected space with respect to an ideal and examine some basic properties. Let $(X, \tau)$ is a topological space and $I$ is an ideal on $X$. A space $X$ is said to be locally $I-$ connected at $x$ if for every open set $V$ containing $x$, there exists a subset $U$ of $X$ containing $x$ such that $U - V \in I$. If all points of space $X$ are locally $I$-connected, this space is called a locally $I$-connected space. We have revealed the main features of this weaker form of locally connectedness. Next It has been shown that it is preserved under continuous functions depending on what conditions. Then we examine the $I-$connected property on the new topology introduced by via ideal. In addition, it was revealed under what conditions locally $I$-connectedness and locally connectedness coincide with one another and under what conditions one differs from another.

Keywords: Locally I-connectedness, Locally connecdeness, Ideal topological space
2020 Mathematics Subject Classification: 54A05, 54D05, 54B05.

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# A Finite Difference Scheme for Singularly Perturbed Neutral Type Differential Equations

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Neutral type delay differential equations arise widely in scientific fields such as biology, ecology, medicine, physics, engineering, etc [1, 2]. Besides, singularly perturbed differential equations are ubiquitous in mathematical problems in the sciences and engineering. Examples include the Navier–Stokes equations of fluid flow at high Reynolds number, the equations governing flow in a porous medium, fluid mechanics, elasticity, quantum mechanics, plasticity, oceanography, meteorology, and mathematical models of liquid materials and of chemical reactions [3].

This study deals with the singularly perturbed initial value problem for a first-order neutral type delay differential equations. For the numerical solution of this problem, a finite difference scheme given on a uniform mesh is presented. The scheme is constructed by the method based on using appropriate quadrature rules with the weight and remainder terms in integral form. Uniform convergence is proved in the discrete maximum norm with respect to perturbation parameter. We formulate the iterative algorithm for solving the discrete problem and present numerical results which validate the theoretical analysis computationally.

Keywords: Singular perturbation, neutral delay differential equation, uniform convergence.

2020 Mathematics Subject Classification: 34K26, 65L05, 65L11.

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# A Numerical Approach for System of Ordinary Differential Equations

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The systems of differential equations appear in many scientific processes such as physics, engineering, chemistry, etc. Especially the system of differential equations arise in modelling of these areas: electrical circuits, population dynamics, feedback control systems [1]. In this study, we deal with a class of the initial value problem for the system of differential equations. Even if there are many analytical methods for systems of differential equations, the exact solution cannot be obtained always. Therefore, the numerical methods gain more importance at this juncture. In recent years, studies on the solution of differential equation systems are quite remarkable. For example, differential transformation method, variational iteration methods, finite difference method were examined ([2, 3, 4] and reference therein). We construct a new difference scheme by the method of integral identities using interpolating quadrature rules with remainder terms in integral form. We prove that the method is convergent in the discrete maximum norm. We present an example to illustrate the theoretical results obtained. The computational results for presented method and Euler method are displayed in tables. By comparing these results, we show that the presented method is more efficient than the Euler method.

Keywords: System of differential equations, finite difference method, error estimates.
2020 Mathematics Subject Classification: 34A30, 65L05, 65L12.

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# Solving Abel's Integral Equation by Kashuri Fundo Transform

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Integral equations can be defined as equations in which unknown function to be determined appears under the integral sign [1, 2]. These equations have been used in many problems occurring in different fields due to the connection they establish with differential equations [3, 4]. Abel's integral equation is an important singular integral equation used in microscopy, seismology, radio astronomy, satellite photometry of airglows, electron emission, atomic scattering, radar ranging, optical fiber evaluation and most commonly flame and plasma diagnostics and X-ray radiography [5]. Integral transforms are widely used mathematical techniques for solving advanced problems of applied sciences. One of these transforms is the Kashuri Fundo transform [6]. This transform was derived by Kashuri and Fundo to facilitate the solution processes of ordinary and partial differential equations. In some works, it has been seen that it provides great convenience in finding the unknown function in integral equations. In this work, our aim is to solve Abel's integral equation by Kashuri Fundo transform and some applications are made to explain the solution procedure of Abel's integral equation by Kashuri Fundo transform.

Keywords: Abel's integral equation, Kashuri Fundo transform, Convolution theorem.
2020 Mathematics Subject Classification: 45E10, 44A15, 44A35.
Acknowledgement: This study is a part of the M.Sc. Thesis of the first author.

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# Existence and uniqueness of positive periodic solutions for a kind of first order neutral functional differential equations with variable delays

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In this work, we present some sufficient criteria for the existence and uniqueness of positive periodic solutions for a class of neutral differential equations inspired by some biological models where these equations involve two types of delays, the first one is a time varying delay and the other depends upon the time and the state variables, leading in turn to the appearance of the second iterate of the state in the production term. The technique used here is based on Banach and Krasnoselskii's fixed point theorems with the Green's functions method. Our results extend and improve previous ones established in the literature.

Keywords: Periodic solution, fixed point theorem, population dynamics.
2020 Mathematics Subject Classification: 35B10, 47H10, 92D25.

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# Under Truncated Random Data, Nonparametric Relative Error Estimation Via Functional Regressor Using The $k$ Nearest Neighbors Smoothing

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In this work, the $k$NN method is used to examine the relationship between a functional random covariate and a scalar answer because of the left truncation by a different random variable. Specifically, we must use mean squared relative error to generate a nonparametric $k$NN regression operator of these functional truncated data as a loss function. In the number of neighbors, we define an estimator and evaluate the uniform consistency performance with the convergence rate. To demonstrate the practicality of our estimate procedure and to highlight its superiority to traditional kernel estimation, a simulation analysis was performed on finitesized samples.

Keywords: Functional data analysis; small ball probability; $k$NN method; local linear method (LLM) consistency; spacial data; almost complete convergence.
2020 Mathematics Subject Classification: First, Second, Third.

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# An existence result for a class of nonconvex second order differential inclusions

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We prove the existence of absolutely continuous solutions to the differential inclusion

$$\ddot{x}(t) \in F(x(t), \dot{x}(t)) + h(t, x(t), \dot{x}(t)), x(0) = x_0, \dot{x}(0) = y_0, \tag{1}$$

where $F$ is an upper semi-continuous set-valued function with compact values such that $F(x, y) \subset \partial f(y)$ on $[0, T]$, where $f$ is a primal lower nice function, and $h$ a single valued Carathéodory perturbation.
Existence results for second order differential inclusions were obtained by many authors, see for instance [1, 4, 5, 6]. The case when $F$ is an upper semi-continuous, compact valued multifunction, such that $F(x, y) \subset \partial f(y)$, for some convex proper lower semi-continuous function $f$, was considered in [3]. We aim at showing that existence of solution of (1) holds in the context of pln lower semi-continuous functions $f$, which is an extension of the result obtained by authors in [2] for the first order problem.

Keywords: differential inclusions, subdifferentials, primal lower nice functions.
2020 Mathematics Subject Classification: 49A52, 49J53, 34A60.

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# Differential equations of divergence form by Topological Degree in Musielak-Orlicz-Sobolev Spaces

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By using the Berkovits degree theory, we prove the existence of at least one solution for the differential equation of divergence form

$$-div\ a_1(x, \nabla u) + a_0(x, u) = f(x, u)$$

with homogeneous Neumann boundary condition in Musielak-Orlicz-Sobolev spaces.

Keywords: Nonlinear elliptic equation, Weak solutions, Musielak-Orlicz-Sobolev space, Topological degree.
2020 Mathematics Subject Classification: 35J66, 47H11, 47J05, 35D30.

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# Immigration and Qualitative Behavior of a Two-Dimensional Discrete-Time Model

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Over the past few centuries, the dynamic behavior of the predator-prey model has been one of the most interesting topics in biological models. Due to the importance of the Lotka-Volterra model, its dynamical properties have been studied over the years. So, in this study, we consider a discrete-time Lotka-Volterra predator-prey model with linear functional response. A constant immigration for prey population is involved to the model. Thus, this two-dimensional discrete-time model has been more realistic. Initially, we deal with the existence conditions for fixed point of considered the model and its stability criterion. Then, by using bifurcation theory, we present bifurcation analysis for the considered model. Finally, we have performed numerical simulations to confirm the accuracy of the theoretical findings by using Matlab software one of the most popular computer algebra system programs.

Keywords: Stability, bifurcation, immigration.
2020 Mathematics Subject Classification: 37G35, 39A30, 39A33.

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# Existence, uniqueness and stability results for a neutral Mackey-Glass type delay differential equation with an iterative production term

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This work deals with a neutral hematopoiesis model with an iterative production term that describes the blood cells production in the bone marrow. Some sufficient conditions are obtained by employing the Krasnoselskii's and Banach fixed point theorems combined with the Green's functions method in order to establish the existence, uniqueness and continuous dependence on parameters of positive periodic solutions. Our theoretical outcomes enrich some existing ones in recent literature.

Keywords: Fixed point theorem, Green's function method, iterative equation.
2020 Mathematics Subject Classification: 47H10, 65M80, 39B12.

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# A numerical approach for a class of singularly perturbed differential-difference equation

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In this paper, we examine the singularly perturbed boundary value problem (SPBVP) for a nonlinear second-order differential-difference equation with mixed type delay. These problems appear in science and engineering fields such as the study of human pupil light reflex, first-exit problems in neurobiology, models of physiological processes and diseases, optimal control theory, models of climate systems, optically bistable devices and signal transmission, quantum photonic systems ([1, 2] and reference therein).

On the other hand, for small values of perturbation parameter $\varepsilon$, standard numerical methods for solving SPBVPs are unstable and do not give accurate results. Therefore, it is important to develop suitable numerical methods for solving these problems, whose accuracy does not depend on the parameter value $\varepsilon$, i.e., methods that are convergent $\varepsilon$-uniformly [3].

Firstly, we present some important properties of the exact solution of the problem. Next, in order to the numerical solution of this problem, we use a fitted difference scheme on a piecewise uniform mesh of Shishkin type which is accomplished by the method of integral identities with the use of linear basis functions and interpolating quadrature rules with weight and remainder term in integral form. It has shown that it gives almost first-order uniform convergence in the discrete maximum norm, independently of the perturbation parameter. Finally, we present the numerical experiments that their results support of the theory.

Keywords: Singular perturbation, differential-difference equation, uniform convergence.
2020 Mathematics Subject Classification: 34K26, 65L10, 65L12.

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# Solving One-Dimensional Bratu's Problem via Kashuri Fundo Decomposition Method

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Nonlinear differential equations are of fundamental importance in science and engineering. The nonlinear Bratu's boundary value problem arises in a large variety of application areas such as solid fuel ignition model of thermal combustion, radiative heat transfer, thermal reaction, electrospinning process for the manufacturing of nanofibers, the Chandrasekhar model of the expansion of the universe, chemical reactor theory and nanotechnology [1, 2, 3]. Due to its simplicity, the Bratu's equation is used as a benchmarking tool for various numerical methods such as finite difference method, finite element approach, weighted residual method, variational iteration method, differential transformation method, homotopy analysis. Some nonlinear equations are difficult to solve analytically. For this reason, new methods are being researched and new studies are carried out in order to find better and newer numerical solutions to nonlinear equations. Various numerical techniques such as one-dimensional differential transformation method, finite difference method, finite element approximation, weighted residual method, shooting method, Laplace Adomian decomposition method have been applied to the Bratu equation. In this study, our aim is to solve nonlinear Bratu's problem via Kashuri Fundo decomposition method [4, 5] which is a combined form of the Kashuri Fundo transform method [6] and the Adomian decomposition method [7, 8].

Keywords: Bratu's boundary value problem, Kashuri Fundo transform, Kashuri Fundo decomposition method.
2020 Mathematics Subject Classification: 34B15, 44A15, 65R10.
Acknowledgement: This study is a part of the M.Sc. Thesis of the second author.

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# Construction of novel analytical solutions of two space-time fractional models with the extended $\left(\frac{G'}{G^2}\right)$-expansion technique

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The extended $\left(\frac{G'}{G^2}\right)$-expansion technique is utilized in this research to provide novel analytical solutions to two nonlinear space-time fractional models in the RLW-class. The fractional derivatives are formed in the conformable sense. The suggested technique generates trigonometric, hyperbolic, and rational wave solutions based on the Riccati differential equation. Mathematica software is used to solve the system of algebraic equations, create graphs, and check the verification of analytical solutions. According to the results, this methodology is a convenient and accurate mathematical tool for solving fractional-order nonlinear equations.

Keywords: The extended $\left(\frac{G'}{G^2}\right)$-expansion method, analytical solution, the fractional differential equations.
2020 Mathematics Subject Classification: 35C07, 35R11, 35Q53.

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# Darboux Frame with Respect to Generalized Fermi-Walker Derivative

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Rest spaces of an observer are transported through Levi-Civita parallelism when the observer $\gamma$ is freely falling. If $\gamma$ is not freely falling, the rest space also is not transported by Levi-Civita parallelism anymore. Therefore, Fermi-Walker derivative was defined for accelerated observers. Then, Pripoae [4,5] enlarged the context by defining a rich class of generalized Fermi-Walker connections which are relevant for both accelerating and non-accelerating observers. In this study, generalized Fermi-Walker derivative, generalized Fermi-Walker parallelism and generalized non-rotating frame are investigated along any curve on any surface in Euclidean space. Initially, we investigate the conditions of the generalized Fermi-Walker parallelism of any vector field along any curve on any surface in Euclidean space by considering the Darboux frame. We show that Darboux frame is generalized non-rotating frame along all curves with the choice of tensor field. We analyse the situation of the generalized Fermi-Walker derivative that coincides with the Fermi-Walker one.

Keywords: Generalized Fermi-Walker derivative, generalized non-rotating frame, Darboux frame.
2020 Mathematics Subject Classification: 53A04, 53B20, 53Z05.

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# Generalized Fermi Derivative on Surfaces in Euclidean 3-Space

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To interpret the universe, it needs to be observed. An observer needs an appropriate frame construction its geometric analysis at a proper time. Rest spaces of an observer are transported through Levi-Civita parallelism when the observer $\gamma$ is freely falling. If $\gamma$ is not freely falling, the rest space also is not transported by Levi-Civita parallelism anymore. From this point of view, Fermi-Walker derivative was defined for accelerated observers. Thereafter, Pripoae [4,5] enlarged the context by defining a rich class of generalized Fermi-Walker connections which are relevant for both accelerating and non-accelerating observers. In this study, generalized Fermi derivative, generalized Fermi parallelism, and generalized non-rotating frame concepts are given along any curve on any hypersurface in $E^{n+1}$ Euclidean space. The generalized Fermi derivative of a vector field and being generalized non-rotating frame conditions are analyzed along the curve on the surface in Euclidean 3-space. Then a correlation is found between generalized Fermi derivative, Fermi derivative, and Levi-Civita derivative in $E^3$.

Keywords: Generalized Fermi derivative, tensor field, surface.
2020 Mathematics Subject Classification: 53A05, 53B20, 53Z05.

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# Generalized Fermi Derivative with Regard to Hypersurfaces

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The universe needs to be observed to make correct interpretations. An observer needs an appropriate frame construction for the definition of its location and its geometric analysis at a proper time. Rest spaces of an observer are transported through Levi-Civita parallelism when the observer $\gamma$ is freely falling. If $\gamma$ is not freely falling, the rest space also is not transported by Levi-Civita parallelism anymore. Thus, Fermi-Walker derivative was defined for accelerated observers. After that, Pripoae [4,5] enlarged the context by defining a rich class of generalized Fermi-Walker connections which are relevant for both accelerating and non-accelerating observers. In this study, generalized Fermi derivative, generalized Fermi parallelism, and generalized non-rotating frame concepts are given along any curve on any hypersurface in $E^{n+1}$ Euclidean space. After that, we examine generalized Fermi parallel vector fields and conditions of being generalized non-rotating frame with the tensor field in $E^4$. Generalizations have been made in $E^n$.

Keywords: Generalized Fermi derivative, generalized non-rotating frame, hypersurface.

2020 Mathematics Subject Classification: 53A05, 53B20, 53Z05.

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# Centered Polygonal Numbers and Polygonal Numbers

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Polygonal numbers are numbers that can be represented by a regular and discrete geometric pattern of evenly spaced points. This study presents a brief explanation on the classifications of polygonal numbers [1, 2]. Then some different ways for covering "centered polygonal numbers, $CS_m(n)$" with "polygonal numbers, $S_m(n)$" are given. These proofs are supported by visual proofs.

Keywords: polygonal numbers, Diophantine equation, visual proofs.
2020 Mathematics Subject Classification: 11A25,11A67,11B75.

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# B-spline method for solving fractional delay differential equations

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In this paper, we used the fractional collocation method based on the B-spline basis to derive the numerical solutions for a special form of fractional delay differential equations (DFDEs). The fractional derivative used is defined in the sense of Caputo. So, we can represent the DFDE under consideration into a matrix form that can be solved using matrix operations and tools from linear algebra. As a result, we get algebraic equations with unknown coefficients that can be solved efficiently using a computer algorithm. To illustrate the validity and efficiency of the method, exact and approximate solutions are compared, and absolute errors are found using different examples. The numerical results, which are backed up by simulation, reveal that the absolute error is very small and that the approach is extremely efficient.

Keywords: Fractional delay differential equation; B-Spline method; Caputo derivative.
2020 Mathematics Subject Classification: 76W05, 76A05, 65L05.

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# Generalized Fibonacci Polynomials Associated with Finite Operators

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Fibonacci numbers and their various generalizations are an extensive area of study that many mathematicians have dealt with for more than a century. There are numerous formulas containing these numbers in the literature. However, finite operators (such as identity operator, the forward difference operator, the backward difference operator, the means operator, Gould operator) have lots of applications in engineering, physics, and applied mathematics. In addition, many researchers in different fields frequently use finite operators in their calculations. Accordingly, the aim of this talk is to show that some properties of the $(p, q)-$Fibonacci finite operator polynomials by implementing the finite operator to the $(p, q)-$Fibonacci polynomials. We derive the Binet-like formula, various generating functions and binomial sum of $(p, q)-$Fibonacci finite operator polynomials. After that we give determinantal expressions for these finite operator polynomials and their special cases. Lastly, we regain, in a different way, recurrence relation for these finite operator polynomials.

Keywords: Finite operators, Generating function, Tridiagonal determinant.
2020 Mathematics Subject Classification: 11B39, 11C08, 11C20, 11Y55.

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# Existence problem for first order evolution inclusion

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In the finite dimensional setting we are interested to the study of the existence of absolutely continuous solutions for a perturbed first order sweeping process differential inclusion governed by the normal cone to a mobile set at point $x(t)$, it has the following form

$$\begin{cases} -\dot{x}(t) \in N_{C_b(t)}(x(t)) + F(t, x(t)), & \text{a.e. } t \in [0, T], \\ x(t) \in C_b(t) \quad for\ all\ t \in [0, T], \\ x(0) = x_0 \in C_b(0), \end{cases}$$

where $C_b(t) = C(t) \cap b\overline{B}$ with $b > 0$, is a moving set time depending with closed convex values of $\mathbb{R}^n$ and $F : [0, T] \times b\overline{B} \rightrightarrows \mathbb{R}^n$ is a multifunction with nonempty, closed, nonconvex values and Lipschitz with respect to the $\rho$-Hausdorff distance with minimal norm satisfies a linear growth condition. In our proof we use a discretization technique based on the catching-up projection algorithm, and we use the basic assumption $\rho$-Hausdorff Lipschitz property of $F(t, x)$ with respect to $x$ and $t$, i.e.,

$$haus_\rho(F(t, x), F(s, y)) < \beta\|x - y\| + \gamma|t - s|,$$

where for any $\rho \geq 0$, $haus_\rho(.,.)$ is the $\rho$-Hausdorff metric on the space of nonempty closed sets, while $\beta, \gamma \geq 0$.

Keywords: Normal cone, sweeping process, perturbation.
2020 Mathematics Subject Classification: 34A60, 28A20, 28A25.

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# Simultaneously Square and Centered Square Numbers Related with Pell and Lucas Numbers

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Polygonal numbers and centered polygonal numbers are positive integer numbers that can be denoted by regular geometric patterns. The theory of polygonal numbers and centered polygonal numbers does not belong to the central domains of mathematics, but the beauty of these numbers has attracted the attention of many scientist for thousands of years, see [1]. One of these scientists is Schlicker. In [2], he proved which numbers are simultaneously polygonal numbers and centered polygonal numbers.
In this talk, we aim to inform the audience about definition and some properties of polygonal numbers and centered polygonal numbers. In particular, we will discuss square numbers and centered square numbers with the pell equation $x^2 - k\,(k-2)\,y^2 = 2k$, for $k = 4$. And finally we studied positive solutions of this equation by using pell and lucas numbers.

Keywords: Polygonal numbers, centered polygonal numbers, pell equation.
2020 Mathematics Subject Classification: 11A25, 11A67, 11D41.

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# The Drazin Inverse for Closed Linear Operators

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In this work, we define an elementary introduction to the Drazin inverse for closed linear operators, give some recent results of Koliha and Trung Dinh Tran, and characterize the different theorems yielded on complex Banach spaces. Furthermore, we present some of the most important characterization of Drazin inverse; in particular, the Drazin invertibility of a closed linear operator T is equivalent to 0 not being an accumulation point of the spectrum of T, also, it is equivalent to T being the direct sum of two operators, where the first one is bounded quasi-nilpotent, and the second one is closed invertible.
The following are some basic concepts of operator theory of closed linear operators we rely on [1]. By $\mathcal{C}(X)$ we denote the space of all closed linear operators $T$ with domain and range in a banach space $X$; $\mathcal{D}(T)$, $\mathcal{N}(T)$ and $\mathcal{R}(T)$ denote the domain, null-space and rang of $T$. If $T \in \mathcal{C}(X)$, then $\rho(T)$ denote the resolvent set of $T$ and $\sigma(T)$ the spectrum of $T$. By $iso\ \sigma(T)$ and $acc\ \sigma(T)$ we define the set of all isolated and accumulation spectral points of $T$.

Keywords: Drazin inverse, closed linear operators.
2020 Mathematics Subject Classification: 65F20, 47A08.

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# Durrmeyer-type generalization of some linear positive operators

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In this talk, we firsly give the historical background of the linear positive operators. Then we consider the Durrmeyer type generalization of the well-known linear and positive operators. After calculating moments and central moments, we mention uniform convergence and approximation properties of these operators. In the last part, we give the rate of convergence by using modulus of continuity, with the help of functions, which belong to Lipschitz class and by the help of Peetre-$\mathcal{K}$ functionals.

Keywords: Modulus of continuity, Lipschitz class, Peetre-$\mathcal{K}$ functional
2020 Mathematics Subject Classification: 41A25, 41A36, 47A58.

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# Local existence of solutions for a quasilinear hyperbolic equation involving the $p-$laplacian operator.

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In this work, we study the value problem related to the quasilinear hyperbolic equation involving the $p-$laplacian operator.
Our purpose in this paper is: firstly to give an existence theorem for local solutions in Sobolev spaces to the problem.
Then, prove of local existence for initial boundary value problems involving the $p-$laplacian operator by using Faedo Galerkin method. Our goal here, is to prove the existence of solutions for a quasilinear hyperbolic equation involvig the $p-$laplacian operator which defined as

$$\Delta_p = div(|\nabla u|^{p-2}\nabla u).$$

Usually $p \leq 1$.

Keywords: Existence, weak solution, sobolev space, $p-$laplacian operator.
2020 Mathematics Subject Classification: 35A01, 35D30, 46E36.

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# A New Generalization Of The Min and Max Matrices

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Matrix theory is widely used in a variety of areas including applied mathematics, computer science, economics, engineering, operations research, statistics, and others. From past to present, different types of matrices have been defined and studied miscellaneous properties of these matrices such as determinant, inverse, norm and factorizations have been studied by mathematicians.
In this talk, we define a new generalization of the min and max matrices. These newly established matrices are the general form of the min and max matrices. Based on this definition, various linear algebra properties such as the determinants, inverses, norms and factorizations are examined of these matrices. Moreover, we define the Hadamard inverse matrices of these newly established matrices and obtain some linear algebra properties of these type of matrices. Finally, in order to verify our theoretical results, we give some numerical examples. These matrices can be used in many branches of mathematics as well as applied sciences, encryption, and image processing.

Keywords: Max Matrix, Min Matrix, Linear Algebra.
2020 Mathematics Subject Classification: 15A15, 15B05, 15A60,

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# Some Fixed Point Results in Soft Fuzzy Metric Spaces

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Consequence of many forms of uncertainties, we can't always use traditional methods to solve some situations. Theory of probability [11], theory of fuzzy sets [14], theory of intuitionistic fuzzy sets [1], and theory of interval mathematics [2] are examples of mathematical methods that can be used to solve these types of problems. However, as [10] points out, these hypotheses have some flaws. The reason behind this is that inadequacy of the parametrization tool of the theories is one of the difficulties. As a result, Molodtsov [10] proposed the the concept of soft set theory as a non-difficult mathematical tool for uncertain situations. The soft set consists of a parametrized family of subsets of universal set. The parameter set could be anything.

Maji et al. [8, 9] have been advancing their soft set theory research. They presented a theoretical study of soft set theory as well as a soft set application in a decision-making situation. Many authors explored soft set theory and its applications in other fields after that [3, 7, 12, 13]. In addition, Das and Samanta [4, 5, 6] contributed to this field by proposing the concept of soft metric space, which is based on soft points of soft sets. Afterwards, Erduran et al.[15] defined soft fuzzy metric spaces in terms of soft points by the help of soft t-norm and investigated its topological structure.

On the other hand, Banach fixed point principle, which has applications in different fields such as applied mathematics, mathematical analysis, solving economic problems etc., was established by Banach[17] in 1922. Later on a large number of researchers were executed on a variety of generalizations of this principle. In 1968, using a different contraction condition from Banach, Kannan[18] proved a fixed point theorem. Kannan's fixed point theorem is also significant because Subrahmanyam [16] shown that it characterizes metric completeness. That is, a metric space X is complete if and only if it has a fixed point for every Kannan mapping.

The purpose of this study is to give Banach and Kannan's version of fixed point theorems in soft fuzzy metric spaces. Our results can be used for further fixed point theory studies.

Keywords: fixed point, soft fuzzy metric, soft set.
2020 Mathematics Subject Classification: 47H10, 54H25, 54A40.

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# A generalized exponential expansion method to simulate two third-order KdV-type equations

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Nonlinear evolution equations are an essential mathematical tool for studying a wide range of physical phenomena and engineering. Because of this importance, many mathematical approaches have been established and developed to create their traveling wave solutions. In this paper, we investigate the Gardner equation and the potential KdV equation through a well-known analytical approach, namely the generalized exp(-$\varphi(\xi)$)-expansion method. We found some traveling wave solutions of the above mentioned equations.

Keywords: generalized exp(-$\varphi(\xi)$)-expansion method, nonlinear evolution equation, Gardner equation, potential KdV equation, traveling wave solution. (-$\theta(\vartheta)$)

2020 Mathematics Subject Classification: 35Q53, 35G20, 35C07

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# COMPARATIVE NUMERICAL STUDY BETWEEN LINE SEARCH METHODS AND MINORANT FUNCTIONS IN BARRIER LOGARITHMIC METHODS FOR LINEAR PROGRAMMING

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Interior-point methods are one of the efficient methods developed to solve linear and non linear programming problems. Several algorithms have been proposed to solve the linear programming problem, where, we distinguish three fundamental classes of interior point methods namely: projective interior point methods and their alternatives, central trajectory methods, barrier methods [2]. Our work is based on the latter type of interior point methods for solving linear programming problems. In this work, we propose a logarithmic barrier interior-point method for solving linear programming problems (LP). In fact, the main difficulty to be anticipated in establishing an iteration in such a method will come from the determination and computation of the step-size. This paper presents a comparative numerical study between line search methods and minorant functions to compute the displacement step in barrier logarithmic method for linear programming. This study favourite minorant function on line search which is promoted by numerical experiments.

Keywords: Interior point methods, Line search, Minorant function.
2020 Mathematics Subject Classification: 90C22, 90C51.

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# Local linear estimation of a conditional quantile for randomly censored functional dependant data

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The nonparametric methods are practical ways to deal with the functional data sets. There are nowadays a large number of fields where functional data are collected such as environmetrics, medicine and finance. In the complete data case and when the regressors are of functional type, [1] have introduced a more general and flexible method than the kernel one. It is the so called local modelling approach. Whereas, in the censoring case, we refer to [2].
In many situations we have to deal with dependent functional complete datasets. One of the most popular examples come from the study of a strong mixing through the functional approach proposed by [3] for nonparametric conditional models, they established their pointwise almost complete convergence. On the other hand, [4] are established the asymptotic properties for the functional locally modeled of the conditional cumulative distribution function.
To our knowledge, the almost complete convergence of the local linear estimation of the conditional quantile based on censored and functional dependent data has not been studied in statistical literature. So, in this work, we address this problem. More precisely, we first present a local linear estimator of the conditional quantile, when the sample is an $\alpha$-mixing sequence. After that, we establish the pointwise and the uniform almost complete convergence of the conditional distribution of the conditional distribution. Then, we deduce the almost complete convergence of the conditional quantile estimator. A simulation study is carried out to show the good behaviour of our estimator.
Keywords: Censored data, Fonctional data, Dependant data.

2020 Mathematics Subject Classification: 62G05, 62G20, 62G99.

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# Modelling of Pancreatic Beta-cells with Gap-junction

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Insulin is secreted from pancreatic beta-cells located on the islets of Langerhans in response to elevated plasma glucose concentrations. Pancreatic beta-cells are electrically excitable cells and insulin secretion results from this electrical activity. Insulin secretion from pancreas is oscillatory. Studies show that insulin oscillations have physiological importance, where systemic insulin oscillations increase the potency of the insulin signal. Furthermore, oscillatory insulin secretion is impaired in patients with type 2 diabetes and their close relatives. Oscillatory insulin secretion results from bursting electrical activity of pancreatic beta-cells and their intra- and inter-islet synchronization. The mechanism for intra-islet synchronization has not been identified yet but the inter-islet synchronization results from gap-junctional coupling between beta-cells within the same islet. Gap junctions on pancreatic beta-cell membranes are comprised of connexin36 (cx36) proteins. Studies show that impairments in the expression cx36 gene impairs oscillatory insulin secretion in animal models. In this study, we developed a computational model of a cluster of beta-cells, which are coupled through electrical currents that represent gap-junctional coupling. In the model, the dynamics of beta-cell electrophysiology is represented by a large system of non-linear differential equations. We investigate the role that cx36 plays in the coupling between pancreatic beta-cells. Through mathematical analysis, we explore the way the strength of the electric coupling between pancreatic beta-cells effects the progression of diabetes.

Keywords: nonlinear dynamics, mathematical modeling, electrophysiology, differential equations.

# Some Density Properties in Bitopological Context

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Many of ones defined in classical topology are defined and still kept to be studying in Bitopological Spaces. A bitopological Space (or Bispace) is simply a set $X$ endowed with two different topology like $\tau_i$ and $\tau_j$ and referred to as $(X, \tau_i, \tau_j)$. In this sense, we introduce *somewhere-density* properties by using generalized open sets in bispaces and investigate the properties of them. We later construct set operators based upon this $somewhere-density$ and give the relatios between them. Moreover, we characterize them in hyperspaces and construct a new filter and ideal with this types of somewhere dense sets. Finally, we give new separation axioms properties taking this somewhere dense sets into consideration.

Keywords: Bitopology, somewhere-density, hyperspaces
2020 Mathematics Subject Classification: 54E55.

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# On integral bases and monogeneity of certain pure number fields defined by $x^{p^r} - a$

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Let $K = \mathbb{Q}(\alpha)$ be a pure number field generated by a complex root $\alpha$ of a monic irreducible polynomial $x^{p^r} - a \in \mathbb{Z}[x]$, where $p$ is a rational prime integer and $r$ is a positive integer. Let $\mathbb{Z}_K$ be the ring of integers of $K$. In this paper, we calculate an integral basis of $\mathbb{Z}_K$ and we study the monogenity of $K$ in some particular cases.

Keywords: Theorem of Dedekind, Theorem of Ore, prime ideal factorization, Newton polygon, Index of a number field, Power integral basis, Monogenic.
2020 Mathematics Subject Classification: 11R04, 11Y40, 11R21.

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# Mathematical Model of COVID-19 with Imperfect Vaccine and Virus Mutation

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This study examines the effect of a partially protective vaccine on COVID-19 infection with the original and mutant virus with the help of a deterministic mathematical model developed. The model we developed consists of six compartments and fifteen parameters. The model consists of S (susceptible = susceptible), $V$ (vaccinated = vaccinated), $E$ (exposed = exposed), $I_1$ (infected with original virus), $I_2$ (infected with mutant virus) and $R$ (recover = recovered) subcompartments. With the established model, we examined the effect of defective vaccine and mutant virus on COVID-19. We considered the incubation period of the disease in the model. We achieved this by handling the $E$ (exposed) class in the model. We examined the effect of both artificial active immunity (vaccinated) and natural active immunity (passing disease) in the model. Since it is known that the recovery and death rates of the original virus and the mutant virus are different in COVID-19, we considered this situation in the study. Then, we performed local stability analysis by calculating the disease-free equilibrium point and endemic equilibrium point of the model. We also obtained the basic reproduction number with the help of the next generation matrix method.

Keywords: $SVEI_1I_2R$ model, vaccine and mutation, stability, basic reproduction number, deterministic model.
2020 Mathematics Subject Classification: 92D30, 00A71, 34D20.

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# Existence result of a Capacity Solution for a Nonlinear Parabolic-Elliptic System

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In this work, we study a much more general version of a thermistor problem than the one considered by Xu; indeed, we assume that the diffusion function $a(x, t, u, \nabla u)$ depends also on u and define a Leray–Lions operator of order $p \geq 2$. Since capacity solutions are obtained by approximating techniques, the proof of the existence theorem relies on the introduction of a sequence of approximate problems. Then, it is shown that the sequence of solutions to these smooth problems converge (up to a subsequence) in a certain sense to a capacity solution. As a consequence, we get the existence and uniqueness of a capacity solution to a coupled nonlinear parabolic–ellipticin a Sobolev Lebesgue spaces.

Keywords: Thermistor problem, Sobolev Lebesgue spaces, Capacity solution.
2020 Mathematics Subject Classification: ..., 46E35, 31A15.

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# Some fixed point theorems for a Generalized cyclic $(\alpha, f, \phi, \psi)$ −contractive mapping in $b$−Metric-Like Spaces

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Our contribution of this work is to introduce a new type of generalized cyclic contractions using the concept of C-class functions in the context b-metric-like spaces. On the other hand, we prove a general fixed point theorem for mappings satisfying the cyclical contractive condition, which extends several results from the literature.

Keywords: -admissible mapping, C-function, cyclic contraction, fixed point, metric like spaces.
2020 Mathematics Subject Classification: MSC 47H10,54H25.

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# Analyzing Neimark-Sacker Bifurcation and Stability for a Discrete-Time Prey- Predator Model with Allee Effect

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In nature, prey-predator relationship is a very important population phenomenon that occurs. In population dynamics, when the population density is very low, there is a positive correlation between the population unit growth rate and the popu lation density. This phenomenon has been called the Allee effect [1], since Allee's research [2]. Factors such as mating difficulty, mating depression, food problem, and protection from predator are considered as Allee effect. Analysis of systems involving Allee effect has gained lots of importance in problems associated with various fields such as conservation biology [3], sustainable harvesting [4], pest control, biological control [5], population management [6], biological invasions [7], interacting species [8]. Therefore, studies on Allee effect have received more and more attention from both mathematicians and ecologists.

In this study, a discrete time predator-prey model with Allee effect in prey population is considered. Using the Euler scheme method to the continuous model in [9], the discrete-time model has been obtained. Then, the existence of the fixed points and their topological classiffications are analyzed algebraically. Also, the direction of Neimark-Sacker bifurcation has been given. The OGY method has been applied in order to control chaos in considered model due to emergence of Neimark-Sacker bifurcation In addition the chaotic features have been justified numerically by computing Lyapunov exponents.

Keywords: Prey-predator model, Stability analysis, Fixed point, Allee effect, Neimark-Sacker bifurcation, chaos control

2020 Mathematics Subject Classification: 39A33, 37G35, 39A30.

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# On the relationship between the degree of coefficients and the growth of solutions of ultrametric $q$-difference equations

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Let $\mathbb{K}$ be an algebraically closed field complete for an ultrametric absolute value. We denote by $\mathcal{M}(\mathbb{K})$ the field of meromorphic functions in $\mathbb{K}$ and $\mathcal{A}(\mathbb{K})$ the $\mathbb{K}$-algebra of entire function in $\mathbb{K}$. By the ultrametric Nevanlinna theory, we investigate the growth of transcendantal meromorphic solutions of some ultrametric $q$-difference equations. These equations arise from the analogue study of $q$-difference equations of Schröder type, we give also somme characterizations of the order of growth for transcendantal meromorphic solutions of the following equation

$$\sum_{j=1}^{n} A_j(x)f(q^j x) = R(x, f(x)) = \frac{\sum_{i=0}^{p} B_i(x)f(x)^i}{\sum_{i=0}^{t} C_i(x)f(x)^i}, \tag{1}$$

where $A_0(x), ..., A_n(x), B_0(x), B_1(x), ..., B_p(x), C_0(x), C_1(x), ..., C_t(x)$ are rational functions in all $\mathbb{K}$ such that $B_p(x)C_t(x) \neq 0$, and $q \in \mathbb{K}$ such that $|q| > 1$.

Keywords: The growth order, Ultrametric meromorphic function, Nevanlinna theory.
2020 Mathematics Subject Classification: 12J25, 32A22, 39A13.

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# Numerical Solution of simple mechanical systems with deep learning

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Ordinary differential equations that explain the time-dependent evolution of mechanical systems have been solved using deep learning methods[1]. We present a method to solve initial value problems using deep learning with energy preserved loss for simple mechanical systems. Numerical symplectic integral methods cannot conserve energy. Conversely, numerical methods conserving energy cannot preserve the symplectic structure[2]. For this, we first solve ODEs using symplectic Euler method. Here, we obtain a solution against time data to use in deep learning training. Namely, the symplectic Euler method is utilized to collect data points, which are then combined into a dataset. Then we improve the solution of the symplectic Euler method using deep learning with energy preserved loss for preserving energy on this dataset. In conclusion we have preserved both the symplectic structure and energy. Moreover, the obtained solution using deep learning is not discrete but continuous.

Keywords: Simple mechanical systems, deep learning, artificial neural networks, ODEs, supervised learning.
2020 Mathematics Subject Classification: 70H20, 37M15, 65Lxx, 65Kxx.

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# Uniqueness of solution of an inverse problem for the ultrahyperbolic Schrödinger equation

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We consider an inverse problem for the ultrahyperbolic Schrödinger equation with variable coefficients. Ultrahyperbolic Schrödinger equations have important applications in [2, 12, 13]. We study the uniqueness of the solution of the problem. We obtain a pointwise Carleman inequality and then prove the uniqueness theorem by the method proposed in [1, 10]. For this aim, we first reduce the inverse problem to a Cauchy problem for a system of integro-differential equations. Similar inverse problem for second order equations were studied in [9].

In the theory of inverse problems, Carleman estimates were firstly introduced by A. L. Bukhgeim and M. V. Klibanov in [4]. Inverse problems for ultrahyperbolic equations were considered in [1, 10], where the unique continuation and stability were proved by using the Carleman estimates. Gölgeleyen and Yamamoto [6] proved conditional Hölder stability for some inverse problems for ultrahyperbolic equation.

For the classical Schrödinger equation, Baudouin and Puel in [11] established a global Carleman estimate and proved the uniqueness and Lipschitz stability. We also refer [3, 8]. On the other hand, there have been limited number of studies on direct and inverse problems for the ultrahyperbolic Schrödinger equation. The Cauchy problem was considered in [5] and and an inverse problem was studied in [7].

Keywords: Ultrahyperbolic Schrödinger equation, inverse problem, uniqueness theorem, pointwise Carleman inequality.
2020 Mathematics Subject Classification: 35Q40, 35Q41, 35R45.

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# Complexity analysis of a primal-dual interior-point method for convex quadratic optimization based on a new hyperbolic kernel function

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**Abstract.**
Kernel functions play an important role in defining new search directions and analysis of primal-dual interior-point algorithms for convex quadratic optimization problems. In this work, we consider a primal-dual interior-point method for solving convex quadratic optimization problems based on a new kernel function with a hyperbolic barrier term. The iteration complexity of the algorithm is evaluated by using some simple analysis tools and several properties of this kernel function, we prove that our algorithm has $\mathbf{O}\left(\sqrt{n}\log n \log \frac{n}{\epsilon}\right)$ iteration bound for large-update method, which is the best-known complexity bound. Finally, we present some concluding remarks and suggestions for future work.

Keywords: Convex quadratic optimization, Hyperbolic barrier term, Large-update method.
2020 Mathematics Subject Classification: 90C20, 90C25, 90C51.

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# Examining the perceptions of anatolian vocational high school students on mathematics through metaphors

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Based on the idea that the main purpose of mathematics is to produce solutions to problems encountered in real life through critical thinking, establishing relationships and reasoning [1], it can be stated that people should use mathematics effectively in order to continue their lives [2]. The basis of being able to use mathematics is to be able to learn mathematics [3]. In this context, it is thought that perceptions about mathematics have a significant effect on learning mathematics [4]. Metaphors are the most effective tool in examining people's perceptions [5]. Based on this idea, the main purpose of the research is to reveal the perceptions of students studying at Anatolian vocational high schools about mathematics through metaphors. The participants of the research are 60 students studying in the 10th and 11th grades of an Anatolian Vocational High School located in the city center of Erzincan in the fall semester of the 2020-2021 academic year. In order to determine the perceptions of the participants about mathematics within the scope of the research, they were asked to complete the sentence "Mathematics is like............; Because ......................" with appropriate expressions. The research was carried out with the case study, which is one of the qualitative research methods. The data obtained in the research were analyzed by content analysis and interpreted using descriptive statistics. Based on the findings of the study, it was determined that students' perceptions of mathematics were generally positive, and that they saw mathematics as a necessity for the continuation of life and valued mathematics. In other words, the students demonstrated that In addition, although mathematics is a need for students, it has been determined that students associate with complex and difficult concepts and produce metaphors for this. Keywords: Mathematics, metaphor, high school students.

Keywords: Keyword one, keyword two, keyword three.
2020 Mathematics Subject Classification: First, Second, Third.

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# Relative cohomology spaces for some $\mathfrak{osp}(n|2)$-modules.

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In this work, we describe the spaces $\mathrm{H}^k(\mathfrak{osp}(n|2), \mathfrak{so}(n), M)$ for a natural class of $\mathfrak{osp}(n|2)$-modules $M$, and for $n \neq 2$. The Lie superalgebra $\mathfrak{osp}(n|2)$ can be realized as a superalgebra of vector fields on the superline $\mathbb{R}^{1|n}$, this yields to canonical actions on spaces of densities and differential operators on the superline. This result gives the zero, first and second cohomology spaces for these modules of densities and differential operators.

Keywords:Orthosymplectic superalgebras, Modules of differential operators, Cohomologies.
2020 Mathematics Subject Classification: 17B56, 53D55, 58A50, 58H15.

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# FIXED-POINT THEOREMS IN EXTENDED FUZZY METRIC SPACES VIA SOME FUZZY CONTRACTIVE MAPPINGS

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In this article we would like to present a new type of fuzzy contractive mappings which are called $\alpha - \phi - \mathcal{M}^0$ fuzzy contractive and $\beta - \psi - \mathcal{M}^0$ fuzzy contractive,and then we demonstrate two theorems which ensure the existence of a fixed point for these two types of mappings. And so we combine and generalize some existing notions in the literature ([5], [7]). Proved these theorems in the extended fuzzy metric spaces are in the more general version than the existing in the literature ones. In fact, we define new notions which are generalized versions of fuzzy contractive mappings introduced by D. Gopal and C.Vetro [5]. We study these contractions in extended fuzzy metric spaces introduced by V. Gregori et al.[7].

Keywords: Fixed-point; Extended fuzzy metric space, Fuzzy contractive mapping.

2020 Mathematics Subject Classification: 47H10, 03E72, 54E50, 54H25,

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# Solvability of an Inverse Problem for a Kinetic Equation on a Riemannian Manifold

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We consider an inverse problem for a kinetic equation on a Riemannian manifold. Kinetic equations can be used to model a variety of phenomena in different areas, ranging from rarefied gas dynamics and plasma physics to biology and socio-economy, [3].

Solvability of inverse problems for the kinetic equation was studied in [1, 2]. As for the numerical algorithms we refer [4, 5]. The physical meaning of these problems consists in finding particle interaction forces, scattering indicatrices, radiation sources and other physical quantities. Moreover, they are closely interrelated with the integral geometry problems. Namely, many problems of integral geometry are equivalent to the corresponding inverse problems for kinetic equations, and vice versa.

In this work, we investigate the uniqueness of solution of the inverse problem. By using the Fourier analysis, we first reduce the problem to a Cauchy problem for a system of equations. Then, in the Riemannian Coordinates, we prove our main result.

Keywords: Kinetic equation, inverse problem, solvability.
2020 Mathematics Subject Classification: 35A02, 35R10, 82B40.

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# A finite difference method based on the operator for the numerical solution of an inverse source problem backward in time

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In this study, we deal with the abstract inverse source problem governed by a linear differential equation backward in time

$$\begin{cases} \frac{du}{dt} - \mathscr{A}u = p + f(t),\ t \in (0,1)\,, \\ \\ u(1) = \psi,\ u(T_1) = \varphi,\ T_1 \in [0,1)\,, \end{cases} \tag{1}$$

in a Banach space $E$. Here, $(u(t), p)$ is the solution pair of the given problem, and $\mathscr{A}$ is a linear operator such that $-\mathscr{A}$ is the generator of the analytic semigroup $exp\{-t\mathscr{A}\}$ associated with an exponentially decreasing norm. Note that when $T_1 \in (0,1)$, problem (1) becomes an inverse problem of simultaneous recovery of the source $p$ and the initial condition $u(0)$.

In paper [1, Remark 3.2], the well-posedness of abstract inverse source problem (1) was presented. On the other hand, a first order of accuracy explicit difference scheme was proposed in paper [2] for the numerical solution of this problem. The main aims of this study are to propose a finite difference method based on the operator $\mathscr{A}$ for the numerical solution of problem (1), and to give a mathematical and numerical analysis to the method proposed.

Considering some particular forms of the operator $\mathscr{A}$ in proper spaces, the difference scheme proposed for problem (1) and the results obtained for that are extended to some difference schemes for some inverse source problems governed by linear parabolic equations backward in time. Furthermore, an elaborate numerical analysis is carried out by performing the method proposed on several test problems.

Keywords: Inverse problem, numerical solution, stability.
2020 Mathematics Subject Classification: 65J22, 65M32, 65N21.

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# Groups whose proper subgroups of infinite rank are hypercentral-by-finite

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A group $G$ is said to be of finite rank $r$ if every finitely generated subgroup of $G$ can be generated by at most $r$ elements, and $r$ is the least positive integer with a such property. If there is no a such $r$, then the group $G$ is said to be of infinite rank. In recent years, many authors studied the structure of locally (soluble-by-finite) groups $G$ of infinite rank in which every proper subgroup of infinite rank belongs to a given class $\mathfrak{Y}$ and they proved that all proper subgroups of $G$ belong to $\mathfrak{Y}$, sometimes the group $G$ itself belongs to $\mathfrak{Y}$ (see for instance, [1] and [2]). In particular, it is proved in [2, Theorem B'], that an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are locally nilpotent is itself locally nilpotent, where $\mathfrak{X}$ is the class introduced in [3] as the class obtained by taking the closure of the class of periodic locally graded groups by the closure operations $\acute{P}$, $\grave{P}$, and $L$. Clearly $\mathfrak{X}$ is a subclass of the class of locally graded groups that contains all locally (soluble-by-finite) groups and which is $R$-closed. Recall that a group is said to be locally graded if every non-trivial finitely generated subgroup contains a proper subgroup of finite index. In [3], it is proved that an $\mathfrak{X}$-group of finite rank is almost locally soluble. Using [2, Theorem B'] and the fact that locally nilpotent groups of finite rank are hypercentral, one can see that an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are hypercentral has all its proper subgroups hypercentral. In the present work, we consider this problem for the class of hypercentral-by-finite groups and we prove that if $G$ is an $\mathfrak{X}$-group of infinite rank whose proper subgroups of infinite rank are hypercentral-by-finite groups, then so are all proper subgroups of $G$.

Keywords: hypercentral-by-finite, Locally (soluble-by-finite), rank.
2020 Mathematics Subject Classification: 20F19; 20F99.

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# Examination of Mathematics Questions in Secondary Education Transition Exam According to Revised Bloom Taxonomy and Middle School Mathematics Curriculum Objectives

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In the study, it is aimed to determine the relationship between the mathematics questions asked in the secondary education transition examination (STE) and the 8th grade objectives of the middle education mathematics curriculum and to reveal the level of cognitive processes measured according to the revised Bloom taxonomy with the questions. Within the scope of the study, 20 mathematics questions in the 2021 STE and 52 objectives in the 8th grade based on the formation of these questions [1] were analyzed by document analysis, one of the qualitative research methods. The questions and objectives were classified independently by two researchers; descriptive statistics were used in the analysis of the data. While the 2021 STE mathematics questions are operational and metacognitive according to the knowledge dimension of the revised Bloom taxonomy, it has been determined that the 8th grade objectives in mathematics, which are based on the formation of these questions, are factual, conceptual and operational. However, it was concluded that the majority of the 2021 STE mathematics questions measure the cognitive processes in the application and analysis levels in the cognitive dimension of the revised Bloom taxonomy, while the 8th grade mathematics objectives are centered on the understanding and application levels.

Keywords:Mathematics questions in secondary education transition exam, Revised Bloom taxonomy, Middle school mathematics curriculum objectives.

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## Generalized Spherical Fuzzy Hamacher Aggregation Operators

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The aggregation functions have become a significant area of research in recent studies. Motivation for the importance of this topic is given by the fact that the necessity of merging the information contained in a collection of pieces of information into a single one, especially in applied sciences. In this study, we first define some algebraic operations based on Hamacher's t-norm and tconorm in the generalized spherical fuzzy environment and study some fundamental properties. Then we define some generalized spherical fuzzy Hamacher aggregation operators such as generalized spherical fuzzy Hamacher (ordered) weighted averaging (GSHWA/GSHOWA), generalized spherical fuzzy Hamacher hybrid weighted averaging (GSHHWA), generalized spherical fuzzy Hamacher (ordered) weighted geometric (GSHWG/GSHOWG) and generalized spherical fuzzy Hamacher hybrid weighted geometric (GSHHWG) operators. We also show the relationships between all defined operators. Finally, we establish a model for these aggregation operators to find the best solution for the multiple attribute group decision-making problems.

Keywords: Aggregation operators, Hamacher operations, generalized spherical fuzzy set, multi-criteria group decision-making.
2020 Mathematics Subject Classification: 03B52, 90B50, 91B06.

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# Inquiry-Based Learning: A Bibliometric Analysis

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The aim of this study is to analyze 618 articles in the Scopus database between 1990-2019 in the field of inquiry-based learning with the bibliometric analysis method. In this context, using the key concept of "inquiry-based learning" in Scopus databases, all relevant data were taken from the Scopus database, various analyzes were carried out with the R-Studio program, and all bibliometric data of the studies were reached. As a result of the analyzes, the number of articles for the years with the specified boundaries, the annual average number of citations, the most published journals and prominent authors, the citation burst values of the authors, the countries and collaborations of the responsible authors, the most cited articles, word cloud and word tree structures In addition, common citation and cooperation networks were examined under the sub-titles. According to the results obtained, it can be said that the interest in the subject area has increased after 2009. It was determined that most of the articles were published in the journal named PRIMUS, and the authors who made the most studies were B. Panijpan and P. Ruenwongsa. It was determined that the most cited study was the article written by Schraw, Crippen, and Hartley in 2006. It has been determined that the countries most open to cooperation in the articles written are Germany and Finland, and the most used words in the summary and keyword analyzes of the studies are "education", "learning" and "students". In the light of the results obtained, it is believed that this study will guide researchers focused on inquiry-based learning.

Keywords: Inquiry-based learning, bibliometric analysis, R-Studio.

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# Homotopy and Descriptive Homotopy in Computational Proximity

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Computational proximity provides a framework useful in studying and analyzing digital images or video frames. We consider a digital image as a collection of vertexes and endow it with a Cech-like nearness relation on the collection of its subsets, which results in a structured set denoted by a pair $(X, \delta)$. This nearness relation provides some certain properties in such a way as to characterize the similarity of subsets by looking at either their spatially nearness or descriptively nearness. This facilitates the investigation of subsets of $X$ that are either close to or apart from each other. To benefit from combinatorial objects in topology, one can also consider and adapt the traditional forms of algebraic topology [1] to the computational proximity. One tool is a (descriptive) proximal homotopy which is a family of (descriptive) proximally continuous maps parametrized by the unit interval [2]. Then computational proximity together with topology are useful in detecting, analyzing and clustering triangulated single video frame image shapes as well as comparing and contrasting shapes in sequences of triangulated video frames. This talk will be based on proximal homotopy and applications in computational proximity.

Keywords: Combinatorial Homotopy, Proximity, Shape, Video Frame.
2020 Mathematics Subject Classification: 54E05, 55P57.

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# Examination of Preservice Mathematics Teachers' Written Expression Skills for Geometric Objects: Student Diaries

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Developed countries have seen student diaries as a means of making written communication a part of mathematics learning, describing mathematical ideas, clarifying the areas that need improvement and developing positive attitudes towards mathematics [1]. The written expression skills of preservice mathematics teachers on the subject of geometric objects within the scope of geometry learning were examined under the subtitles of being able to define, using concepts and using mathematical language [2]. It was aimed to investigate the possible relationship between preservice teachers' written expression skills and their academic achievements. This study, which is based on the qualitative research approach, was designed with the case study model, since an existing situation was tried to be described in its own conditions. This study was carried out at the 2021-2022 academic year. The participants of the study consisted of preservice teachers studying in the primary school mathematics teaching department of a state university in the Central Anatolia Region. In the research, qualitative data obtained from student diaries were analyzed by content analysis method. When the findings of written expression skills were examined, it was determined that the candidates mostly preferred to use verbal expressions and were weak in using mathematical language. On the other hand, it was seen that the candidates were able to associate the related concept with daily life.

Keywords: Student diaries, written expression skills, geometric objects.
2020 Mathematics Subject Classification: 97D30, 97D40, 97D60, 97G80.

This study was produced from the master thesis of the first author under the supervision of the second author.

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# Examination of Preservice Teachers' Mathematical Thinking and Modeling Skills

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For the solution of the problems in our lives, individuals who value mathematics and who can use mathematics with advanced mathematical thinking power in modeling and problem solving are needed [1]. In order for students to have mathematical thinking and modeling skills, it is thought that teachers should have these skills first. For this reason, it was aimed to determine preservice teachers' mathematical thinking skills (customizing, generalizing, making assumptions, proving) and modeling skills in this study. The case study method was adopted as it was aimed to determine preservice teachers' existing mathematical thinking and modeling skills. In the study, mathematics and classroom teacher candidates studying at a state university in the Central Anatolia Region in the 2021-2022 academic year were studied. In this process, a mathematical modeling skill test was prepared in the context of mathematical thinking. During the development process of the test, the components specified by [2] were taken into account and it was ensured that each modeling problem overlapped with at least one of these components. Then, with the help of this test, it was aimed to examine the mathematical modeling skills of the candidates in the context of mathematical thinking and the test was applied to the pre-service mathematics and classroom teachers. The analysis process of the data continues and the findings will be discussed with the relevant literature.

This study was produced from the master thesis of the first author under the supervision of the second and third authors. Keywords: Mathematical Modeling, Mathematical Modeling Skill, Mathematical Thinking.

2020 Mathematics Subject Classification: First, Second, Third.

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# Concept Images and Misconceptions of Preservice Mathematics Teachers about the Angles and Triangles Concepts

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The aim of this study is to determine the concept images and misconceptions of pre-service mathematics teachers about triangles. In this context, a measurement tool was prepared by scanning the relevant literature and applied to pre-service teachers studying in the department of mathematics education. The qualitative part of this study, which is based on the mixed research approach, was designed with a case study model, and the quantitative part was designed with a relational model. This study was carried out with the participation of teacher candidates studying at a state university in the 2021-2022 academic year. Data collection tools developed by Kaya (2018) (EK1) and Şengül-Akdemir (2017) (EK2) were used to determine the concept images and misconceptions about triangles, which is the aim of the research. In this study, SPSS statistical program and percentage and frequency calculations were used for the analysis of quantitative data. Content analysis was used in qualitative data analysis. Kendall W fit coefficient was calculated in order to determine whether the codes under the revealed category represent the relevant category and the categories represent the relevant theme. As a result of the research, the concept of angle is defined as "the region between the opening formed by the convergence of two rays with the same starting point". In addition, some of the candidates thought that the angle was "the measure of the region between two lines".

Keywords: Concept image, triangles, prospective teachers
2020 Mathematics Subject Classification: First, Second, Third.

Bu çalışma, ikinci yazar danışmanlığında birinci yazarın hazırladığı yüksek lisans tezinden üretilmiştir.

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# On the cosine curve as 4th and 6th order Bézier curve in $E^2$

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In this study we have examined the way how the trigonometric cosine curve can be written as any order Bézier curve. We know that any order Bézier curve is polynomial curve in $E^2$. As a result we need a new method. Taylor series of a function is an infinite sum of terms of the functions derivatives at a single point a, also a Maclaurin series is a taylor series where a=0, hence using the Taylor series and Maclaurin series, first we have examined cosine curve as the 4th order Bézier curve and then the 6th order Bézier curve based on the Bézier curve control points with matrix form in $E^2$. We give the control points of the 4th and the 6th order Bézier curve based on the coefficients of Maclaurin expension for cosine function . Also we give the coefficients of Maclaurin expension for cosine function based on the control points of the 4th and the 6th order Bézier curves in $E^2$.

Keywords: Cosine curve, Maclaurin series, 4th order Bézier curve, 6th order Bézier curve.
2020 Mathematics Subject Classification: 53A04-53A05

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# Existence and multiplicity result for general Steklov problem

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In this presentation, we study the existence of solutions for (p, q)-Laplacian Steklov problem with two parameters.
The main result of our research is to prove in different situations the existence and multiplicity of solutions for a (p, q)-Laplacian Steklov problem, using variational methods.

Keywords: (p, q)-Laplacian, Nonlinear boundary conditions, Variational methods.

2020 Mathematics Subject Classification: 58J10, 58J20, 35Jxx, 35J66, 35J50.

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# A Solution Algorithm for An Inverse Problem for the Kinetic Equation which Involves Poisson Bracket

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In this work, we deal with an inverse source problem for a stationary kinetic equation which involves Poisson Bracket and scattering term. Kinetic equation is a partial differential equation which arises in many areas of science and technology such as semiconductors, stochastic dynamical systems, traffic flow, system biology. Numerical solution of some inverse problems for the stationary kinetic and transport equations were studied in [1-3] by using finite difference and Galerkin methods.

In this study, we first discuss the uniqueness and existence results on the problem. Later, we present a numerical algorithm for the solution of the inverse problem which based on a hybrid approximation, that is composed of finite difference method, Lagrange's polynomial interpolation and Newton's Cotes formula. Finally, we test the proposed method by performing several numerical experiments. The obtained results are presented comparatively via graphs and tables. We conclude that the relative error in reconstruction of the unknown functions is sufficiently small.

Keywords: Inverse problem, kinetic equation, finite difference method, Lagrange interpolation, Newton-Cotes formula.

2020 Mathematics Subject Classification: 35R30, 65N21.

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# A CHARACTERIZATION OF OPEN DISTANCE PATTERN UNIFORM CHORDAL GRAPHS AND DISTANCE HEREDITARY GRAPHS

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Given an arbitrary nonempty subset $A$ of vertices in a $(p, q)$-graph $G = (V, E)$, each vertex $u$ in $G$ is associated with the set $f_A^o(u) = \{d(u, v) : v \in A, u \neq v\}$ , where $d(x, y)$ denotes the usual distance between the vertices $x$ and$y$ in $G$, called its open $A$-distance pattern. $G$ is called an open distance-pattern uniform (or, in short, odpu)-graph if there exists a nonempty subset $A \subseteq V(G)$ such that $f_A^o(u)$ is independent of the choice of $u \in V(G)$ , where the set-valued function (or, set-valuation) $f_A^o$ is called the open distance pattern uniform (or, an odpu-) labeling of $G$and $A$is called an odpu-set of $G$. The minimum cardinality of an odpu-set in, if it exists, is the odpu-number $\varsigma(G)$ of $G$. Given any property $P$, we establish characterization of odpu-graph with property $P$. In this paper, we characterize odpu-chordal graphs and thereby characterize interval graphs, split graphs, strongly chordal graphs and ptolemaic graphs that are odpu-graphs. We also characterize odpu-distance hereditary graphs.

Keywords: Open distance-pattern uniform graphs, Open distancepattern uniform (odpu-) set, Odpu-number, odpu-chordal graphs, odpu-interval graphs, odpu-split graphs, odpu-strongly chordal graphs, odpu-ptolemaic graphs, odpu-distance hereditary graphs.
2020 Mathematics Subject Classification: 05C12

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# Associated curves of a framed curve in Euclidean 3-space

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In this paper, we define some new curves associated to a framed curve in Euclidean 3-space. These new curves which are defined as integral curves of some vector fields of a framed curve includes framed generalized principal-direction curve, framed generalized binormal-direction curve and framed Darboux-direction curve. We obtain relationships between the framed curvatures of new defined framed curves and framed curvatures of given framed curve. By using the obtained relationships we give some characterizations for such curves.

Keywords: framed curve, framed helix, framed slant helix, framed direction curve.

2020 Mathematics Subject Classification: 58K05, 53A04.

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# A New Approach Tubular Surface with a new frame in $G_3$

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A channel or canal surface is a surface produced by the envelope of a family of spheres whose centers are located on the directrix of a space curve. The canal surface is known as a pipe surface when the radius of the producing spheres are constant. The canal surface is called a tube or tubular surface when the radius function r(t) = r is a constant. Tube surfaces are used to show 3-dimensional shapes such as pipes, ropes, poles. In addition, tube surfaces are used for modeling solids and surfaces for computer aided geometric design and fabrication. On the other hand, Galilean geometry is one of the geometries whose motions are the Galilean transformations of classical kinematics. In this paper, first of all, we define a new orthogonal frame in $G_3$. Then, we get condition of general helix in regard to new orthogonal frame in the Galilean 3-space and we obtain characterizations of tubular surfaces with the new orthogonal frame in $G_3$.

Keywords: Tubular surface, helix, Galilean space

2020 Mathematics Subject Classification: 53A04, 53A05, 53A40.

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# Approximate controllability results for Caputo fractional Volterra-Fredholm integro-differential systems of order $1 < r < 2$

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This paper investigates the issue of approximate controllability results for fractional Volterra-Fredholm integro-differential systems of order $r \in (1, 2)$. The main results of this paper are tested by using fractional calculations, cosine families, mild solutions multivalued functions, and Martelli's fixed point theorem. In the beginning, we investigated the approximate controllability results of mild solutions by using sufficient conditions. Finally, an example is presented to illustrate the theory of the obtained result.

Keywords: Fractional differential systems, Cosine and sine families, Mild solutions, Fixed point techniques.
2020 Mathematics Subject Classification: 34A08, 26A33, 34K30, 47D09, 47H10.

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# Parallel Transported Along Dual Lorentzian Spacelike And Timelike Curves

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There are different transport laws such as parallel and Fermi-Walker transport for a tensor along a given curve. The parallel transport for the tensor along the given curve is defined as the law which makes that its covariant derivative be zero. If the curve is a geodesic, then the tangent vector will coincide at another point of the curve with its parallel transported vector. Otherwise, the tangent vector will not coincide with its parallel transported vector. In this case, there is Fermi-Walker's law that another transport law. The Fermi-Walker transport of the tensor along the given curve is defined as the law which makes that its the Fermi-Walker derivative along the curve be zero. In this study, we investigate Fermi-Walker derivative and parallel transported frame along both the non-null unit speed dual Lorentzian timelike curve and the non-null unit speed dual Lorentzian spacelike curve which are with non-null dual principal normals in dual Lorentzian space. Fermi-Walker transport, non-rotating frame and Fermi-Walker termed Darboux vector concepts are given along the non-null unit speed dual Lorentzian curve with non-null dual principal normals. Being Fermi-Walker transport conditions are analyzed for any dual Lorentzian timelike and dual Lorentzian spacelike curves.

Keywords: Fermi-Walker derivative, Fermi-Walker transport, Non-rotating frame.
2020 Mathematics Subject Classification: 53B20, 53B21, 53Z05.

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# Uniform well-posedness and stability for fractional Navier-Stokes equations with Coriolis force in critical Fourier-Besov-Morrey spaces

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In this paper, we study the initial value problem of the three-dimensional fractional Navier-Stokes equations with Coriolis force in critical homogenous Fourier-Besov-Morrey spaces $\mathcal{F}\dot{\mathcal{N}}^{s}_{p,\lambda,q}\left(\mathbb{R}^3\right)$ with $s = 1 - 2\alpha + \frac{3}{p'} + \frac{\lambda}{p}$. By making use of the Fourier localization argument and the Littlewood-Paley theory as in the works $[1, 2, 3, 4, 5]$, we get local well-posedness results and global well-posedness results with small initial data $u_0$, which is a divergence-free vector field, belonging to the critical homogenous Fourier-Besov-Morrey spaces $\mathcal{F}\dot{\mathcal{N}}^{s}_{p,\lambda,q}\left(\mathbb{R}^3\right)$ with $s = 1 - 2\alpha + \frac{3}{p'} + \frac{\lambda}{p}$. Moreover, we prove that the corresponding global solution decays to zero as time goes to infinity, and we give the stability result for global solutions. The space $\mathcal{F}\dot{\mathcal{N}}^{s}_{p,\lambda,q}$ covers many classical spaces, e.g. the Fourier-Herz space $\dot{\mathcal{B}}^{s}_{q}$, the Fourier-Besov-Lebesgue space $F\dot{B}^{s}_{p,\lambda,q}$, and the Lei-Lins space $\chi^{s}$. The result of this paper extends the works of $[1, 2, 3, 4, 5]$.

Keywords: Navier-Stokes equations, global well-posedness, Coriolis force, Fourier-Besov-Morrey space.
2020 Mathematics Subject Classification: 35Q30, 76D05, 76D03.

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# Symmetric functions for $(p,q)$-numbers and Pell Lucas polynomials

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In this work, we give some new generating functions of the products $(p,q)$-Fibonacci numbers, $(p,q)$-Lucas numbers, $(p,q)$-Pell numbers, $(p,q)$-Pell Lucas numbers, $(p,q)$-Jacobsthal numbers and $(p,q)$-Jacobsthal Lucas numbers at positive and negative indices with Pell Lucas polynomials, by making use of useful properties of binary of the symmetric functions.

Keywords: Symmetric functions, Generating functions, Pell Lucas polynomials.
2020 Mathematics Subject Classification: 05E05, 11B39.

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# Existence and uniqueness results for Hilfer fractional integro-differential equation

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In this paper, we discuss the existence and uniqueness of solution for the Hilfer fractional integro-differential equations with nonlocal Erdélyi-Kober fractional condition. First, the equivalence of this class of problem and a nonlinear Volterra integral equation is established. Next, the existence and uniqueness results are obtained by using Krasnoselskii's fixed point theorem. Further, an example is given to illustrate our theory results.

Keywords: Hilfer fractional derivative, integro-differential equations, fixed point theorem.
2020 Mathematics Subject Classification: 26A33, 45G10, 34A12,47H10.

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# On predictors of partial parameters under a partitioned linear model and its reduced models

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Linear regression models are one of the most commonly used statistical methods in the field of statistics and other disciplines. Linear regression models can be divided into certain partitioned forms to conduct statistical inference on partial parameters under the partitioned linear models and some of their reduced models. In this study, we consider a partitioned linear model and its reduced models. We introduce the technical concepts and definitions of consistency and predictability conditions of the best linear unbiased predictors (BLUPs) of unknown vectors including partial parameters. Further, we derive analytical formulas for calculating BLUPs of unknown vectors including partial parameters under a given partitioned linear model and its reduced models. Our main purpose is to derive some results from the comparison of BLUPs under the partitioned linear model and its reduced models by considering covariance matrices of BLUPs. For doing the comparisons, we use the formulas of inertias and ranks of block matrices. The subjects related to the results obtained in this study can also be found in [1-6].

Keywords: BLUP, partitioned linear model, reduced linear model.
2020 Mathematics Subject Classification: 62J05, 62H12, 15A03.

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# Optimal control of a fractional SIR model under the effect of nonlinear incidence and recovery rates

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This study presents optimal control of a fractional SIR epidemic model under the effect of nonlinear incidence and recovery rates. The incidence rate which is a threshold parameter refers to the interaction between those infected and those susceptible for a disease in a population. With the help of this ratio, the course of the disease can be predicted. It can be defined in different ways, taking into account the population size and status of the disease. In this work, nonlinear Monod equation is used as the incidence rate. The analyzed model is considered in terms of the Caputo fractional derivative. The main purpose is to examine the effect of this function in the population after adapting the control function to the model. Control strategy is determined to reduce the number of infected individuals. The optimal system is numerically solved by the Diethelm's predictor-corrector algorithm combined with the forward-backward sweep method. Controlled and uncontrolled behaviors of the system are simulated by MATLAB software. According to the numerical simulations, the reduction in the number of infected individuals is highly desirable under the influence of optimal control.

Keywords: SIR model, optimal control, Caputo fractional derivative, nonlinear incidence rate, nonlinear recovery rate, predictor-corrector algorithm.

2020 Mathematics Subject Classification: 26A33, 93-XX, 49N10.

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# Existence and uniqueness results for a revisited Nicholson's blowflies model with two different variable delays and a nonlinear harvesting term

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The current work is mainly concerned with the existence, uniqueness and stability of positive periodic solutions for a first order delay differential equation that describes the dynamics of a population of Lucila cuprina which is held to be responsible for an estimated 90% of all cutaneous myiasis that cause losses of several hundreds of millions of dollars each year.

This revisited model involves two different variable delays, the first one which represents a harvesting lag is a time varying delay while the second one which denotes the life-cycle duration, depends on both the time and the population of sexually mature adults. By establishing an equivalence between the considered problem and a nonlinear integral equation, and using some properties of the obtained Green's kernel as well as Banach and Schauder's fixed point theorems, we prove our desired results. These last ones are completely new and complement those of previous publications.

Keywords: Fixed point theorem, Green's function, population dynamics
2020 Mathematics Subject Classification: 47H10, 34B27, 92D25.

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# RECURSIVE DOUBLE KERNEL ESTIMATOR OF THE CONDITIONAL QUANTILE FOR FUNCTIONAL ERGODIC DATA

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The main purpose of the present work is to consider the conditional quantile estimator of a scalar response $Y$ given an explanatory variable $X = x$ taking its values in a semi-metric space $\mathcal{H}$. Hence, the asymptotic normality of the proposed estimator is obtained when the observations are sampled from a functional ergodic process. The result confirms the prospect proposed in Benziadi et al. (2016). The usefulness of this result will be illustrated in the construction of confidence bands and as applications, a comparison study based on a finite-sample behavior of the estimator is investigated by simulations as well.

Keywords: Recursive estimate, Conditional quantile, Functional data, Ergodic data, Asymptotic normality.
2020 Mathematics Subject Classification: 62G20, 62G08, 62E20.

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# GENERAL DECAY OF SOLUTIONS IN ONE-DIMENSIONAL POROUS-ELASTIC SYSTEM WITH MEMORY AND DISTRIBUTED DELAY TERM WITH SECOND SOUND

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We investigate a one-dimensional porous-elastic system with the presence of both memory and distributed delay terms in the second equation with second sound. Using the well known energy method combined with Lyapunov functionals approach, we obtain a general decay result.

Keywords: Porous system, General decay, Exponential Decay, Memory term, Distributed delay term.
2020 Mathematics Subject Classification: 35B40, 35L70, 93D15, 93D20.

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# Some new results on periodic solutions for a periodic delay hematopoiesis model with a unimodal production function

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The main purpose of this work is to study a first order hematopoiesis model with periodic coefficients and variable delays. The proofs of the existence results rely essentially on the Schauder's fixed point theorem and the Green's function method. Via the Banach fixed point theorem, the existence, uniqueness and stability of positive periodic solutions are further analyzed. To the best of our knowledge, there are no findings reported in the literature that investigated this problem.

Keywords: Hematopoiesis model, iterative equation, fixed point
2020 Mathematics Subject Classification: 34B27, 47H10.

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# Existence, uniqueness and stability of solutions for a first order iterative functional differential equation

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This work is devoted to investigate a first-order iterative functional differential equation. A set of sufficient conditions for the existence, uniqueness and continuous dependence on parameters of positive periodic solutions are established by the help of the contraction mapping principle as well as some properties of a Green's kernel that is obtained after the conversion of the considered equation into an integral one. Our results are new and generalize some known ones to some extent.

Keywords: Fixed point theorem, iteration, periodic solution.
2020 Mathematics Subject Classification: 47H10, 30D05, 34C25.

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# Some Fixed Point Theorems on O-Complete Metric Spaces

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In this study, some coupled fixed point theorems on metric spaces endowed with an orthogonal relation are presented. In addition, this study consists of an example showing the importance of the results obtained.

Keywords: O-completeness, Fixed Point Theory, Orthogonal Set.
2020 Mathematics Subject Classification: 47H10, 54H25.

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# On Deformed Lifts

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Let $R\left(\varepsilon^2\right)$ be an algebra of order 3 with a canonical basis $\{e_1, e_2, e_3\} = \{1, \varepsilon, \varepsilon^2\}, \varepsilon^3 = 0$. We proved that if $w(z^1, ..., z^n) = y^1(x^1, ..., x^n) + \varepsilon y^2(x^1, ..., x^n) + \varepsilon^2 y^3(x^1, ..., x^n)$, where $z^i = x^i + \varepsilon x^{n+i} + \varepsilon^2 x^{2n+i}, i = 1, ..., n$, is a multi-variable $R(\varepsilon^2)$-holomorphic function, then the function $w = w(z^1, ..., z^n)$ has the following specific form:

$$w\left(z^1, ..., z^n\right) = y^1\left(x^1, ..., x^n\right) + \varepsilon\left(x^{n+i}\partial_i y^1 + G\left(x^1, ..., x^n\right)\right)$$

$$+\varepsilon^2\left(x^{2n+i}\frac{\partial y^1}{\partial x^i} + \frac{1}{2}x^{n+i}x^{n+j}\frac{\partial^2 y^1}{\partial x^i \partial x^j} + x^{n+i}\frac{\partial G}{\partial x^i} + H\left(x^1, ..., x^n\right)\right)$$

Let now $T^2(M_r)$ be the bundle of 2-jets, i.e. the tangent bundle of order 2 over $C^\infty-$ manifold $M_r$, $dimT^2(M_r) = 3r$ and let $(x^i, x^{\bar{i}}, x^{\bar{\bar{i}}})$ be an induced local coordinates in $T^2(M_r)$. We proved that bundle $T^2(M_r)$ is a real modeling of $R(\varepsilon^2)$-holomorphic manifold $X_r(R(\varepsilon^2))$. Using this fact in the bundle of 2-jets we introduce the functions ${}^V f$, ${}^I f$ and ${}^C f$. These functions ${}^V f, {}^I f$ and ${}^C f$ are called recpectively the vertical,intermediate and complete lifts of $f$ in $M_r$ to $T^2(M_r)$. If $g = h = 0$, then we have the 0-th$f^0$, 1-th$f^1$ and 2-th$f^2$ lifts of $f$, i.e. the lifts ${}^I f$ and ${}^C f$ of $f$ to $T^2(M_r)$ are respectively the deformed lifts of 1-th and 2-th lifts of $f$. In the present report some differential-geometrical properties concerning vertical, intermediate and complete lifts were investigated.

Keywords: Holomorphic functions; bundle of 2-jets; deformed lift; 1-forms.
2020 Mathematics Subject Classification: 53C07, 53C15.

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# An examination of the conceptual knowledge of teacher candidates in the elementary mathematics program regarding the concept of ratio

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There are two types of information, namely conceptual and procedural. Conceptual knowledge can be explained as the knowledge of concepts, relations between concepts and definitions in general terms [1]. Procedural knowledge, on the other hand, is the information that includes the processing steps and algorithm used in problem solving [3; 4]. An individual needs to have both types of knowledge to obtain meaningful learning. Even if the individual does not have conceptual knowledge, s/he stil can perform some problem-solving thanks to her/his procedural knowledge. However, it may be difficult for him/her to identify the procedures applied to solve the problem or the relationship between the solution and other problems. Conceptual knowledge gives meaning to and supports procedural knowledge and understanding occurs [2]. That is, a procedural knowledge devoid of a conceptual knowledge background cannot contribute to the mental development of the individual sunstantially. Thus, in the present study, it was aimed to explore the conceptual knowledge of teacher candidates in the elementary mathematics program about the concept of ratio. This qualitative study is designed as a case study and aimed to examine the conceptual knowledge of the teacher candidates in the elementary mathematics program about the concept of ratio in depth. The total number of participants was 105 college students studying in the elementary mathematics teaching program at a state university, and included only 4th-year (73) and 3rd year (32) students. The teacher candidates were asked to define and explain the concept of ratio. The content analysis method was used to analyze the data. As a result of the analysis, 4 basic conceptual structures related to the ratio were found. These were Relation, Comparison, Division and Fraction. The concept of ratio was expressed as the relationship between two or more multiplicities in the relationship category, the comparison of two or more multiplicities in the comparison category, the division of two multiplicities in the division category, and the fraction in the fraction category. The most common views on the concept of ratio were respectively: "(i) Ratio is the comparison of the same or different kinds of multiplicities; (ii) Ratio is the division of two multiplicities of the same or different kind; (iii) Ratio is the relationship between multiplicities of the same or different kind; (iv) Ratio is the relative condition of multiplicities of the same or different kind; (v) Ratio is the relation; and (vi) Ratio is the relation between the part and the whole".

Keywords: Ratio, definition of the concept, conceptual knowledge.

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# Investigating the Impact of Bariatric Surgery on Lipid and Glucose Absorption via Mathematical Modeling

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Obesity has become an important public health problem worldwide because it leads to various comorbidities such as type 2 diabetes (T2D), cardiovascular diseases (CVD), insulin resistance, hypertension, high cholesterol, and cancer. The majority of the people who are struggling with obesity can not succeed at losing weight by conventional methods such as diet and exercise. Moreover, subjects usually regain the weights they lose through these conventional methods in the future. The bariatric surgery, which is an effective method for treating patients with obesity, provides significant weight loss, and results in several metabolic ameliorations. In addition to that, studies show that bariatric surgery is more effective at sustaining the weight loss when compared to conventional methods. Roux-en-Y Gastric Baypass (RYGB) surgery is one of the most commonly performed bariatric surgery methods in the World. With this method, due to the reduced stomach volume, the amount of the consumed food is reduced and some of the nutrients are not absorbed. Therefore, RYGB surgery results in significant weight loss in morbidly obese patients and improvements in liver steatosis, sleep apnea, and insulin resistance. In this study, the impact of the anatomical changes that take place after RYGB surgery on the lipid and glucose absorption has been investigated. For this purpose, we developed physiologically based mathematical models of the gastrointestinal tract and utilized clinical data collected from patients that went through RYGB surgery at Amsterdam University Medical Center. The developed mathematical models are shown to be effective at understanding the impact of the surgery on lipid and glucose absorption. Through mathematical modeling, physiological parameter values were estimated for pre- and post-surgical configurations and their differences were analyzed. This way some important metabolic changes that occur as a result of surgeries have been identified.

Keywords: Obesity, Bariatric Surgery, Mathematical Modeling.
2020 Mathematics Subject Classification: Applied Mathematics, Bio-mathematics, Computational Modeling

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# Jacobi Last Multiplier Method for Optimal Growth model with the Environmental Asset

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This study explores the application of Jacobi last multiplier method to the optimal growth model with the environmental asset. The model is investigated by utilizing the *current* and *present* Hamiltonian functions and determining the first-order conditions for optimal control based on Pontryagin's maximum principle. The main idea of the paper is the analysis of two coupled nonlinear first-order ODEs corresponding to first-order conditions. The method of Jacobi last multiplier (JLM) presents a feasible connection with Lie symmetries. Based on this point of view, Lie symmetries of the optimal growth model with the environmental asset are determined then JLM's are found by using the aforementioned connection. Furthermore, the first integrals of the model are obtained via JLM's.

Keywords: Optimal control problem, Prelle-Singer method, Lie symmetries, Jacobi's last multipliers.
2020 Mathematics Subject Classification: 70G65, 65K10, 34A05.

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# A pointwise Carleman inequality for the general ultrahyperbolic Schrödinger equation

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In this work, we establish a pointwise Carleman inequality for the general ultrahyperbolic Schrödinger equation. The proof is based on the method presented in Amirov [1], Klibanov and Timonov [5] and Lavrentiev et al [6]. A Carleman estimate is a $L^2$-weighted estimate which was first introduced by Torsten Carleman in 1939 as a tool for proving the unique continuation property for elliptic equations.

It is known that classical Schrödinger equation is the master equation in quantum kinetic theory. The Carleman estimates for this equation were obtained by [3, 4, 7] and used for investigation of uniqueness and stability of the solution of the inverse problem.

Here we consider the generalized form of classical Schrödinger equation which is called ultrahyperbolic Schrödinger equation. These equations arise in several applications, for example in water wave problems, [2, 9], in higher dimensions as completely integrable models, see [8]. The obtained inequality in this work can be used to prove the uniqueness and stability results for related direct and inverse problems.

Keywords: Ultrahyperbolic Schrödinger equation, pointwise Carleman inequality, Cauchy problem.
2020 Mathematics Subject Classification: 35Q40, 35Q41, 35R45.

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# An approach to the Diophantine equations with integer sequences

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In this paper, we find all repdigits which can be expressed as the sum of some integer sequences. To prove our main result, we use the combined approach of lower bounds for linear forms in logarithms of algebraic numbers and a version of the Baker Davenport reduction method.

Keywords: Repdigit, integer sequence, Diophantine equation, Baker's theory.
2020 Mathematics Subject Classification: 11B37; 11D45; 11J86.

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# Fekete-Szegö problem for a subclass of bi-univalent functions associated with Gegenbauer polynomials

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In this paper, we introduce and investigate a new subclass of bi-univalent functions defined in the open unit disk, which are associated with the Gegenbauer polynomials. Furthermore, we find estimates for the Taylor-Maclaurin coefficients $|a_2|$, $|a_3|$ and Fekete-Szegö inequality $\left|a_3 - \mu a_2^2\right|$ for functions in this new subclass.

Keywords: Bi-univalent functions, Fekete-Szegö problem, Gegenbauer polynomials.
2020 Mathematics Subject Classification: 30C45, 30C50.

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# Bibliometric Analysis of Scientific Studies on "Noticing Skill" in Mathematics Education

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The systematic evaluation of articles published in academic journals is useful for examining the current status of mathematics education research and identifying future trends [1]. Bibliometrics deals with the statistical analysis of scientific studies, data such as author, subject, cited author, cited sources, and makes it possible to reveal the general structure of a particular discipline in line with the statistical results obtained. These methods have the potential to deliver systematic, transparent and reproducible research, thereby improving the quality of research. Bibliometric methods direct the researcher to the most influential publications [3]. In this study, a bibliometric analysis of published scientific studies on the ability to notice was made. 128 publications with SSCI index were examined from the WoS database. According to the findings, the most broadcasting countries are "United States of America", "Turkey" and "Germany"; the most published journals are "journal of mathematics teacher education", "international journal of science and mathematics education" and "zdm-mathematics education"; It has been seen that the universities with the most publications are "Hamburg University", "Michigan State University" and "Northwestern University". In addition, the cognitive structure related to the ability to notice was revealed and presented visually.

Keywords:Bibliometric Analysis, Noticing, Mathematics Education

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# Hill-type estimator of the tail index for randomly censored heavy-tailed data: Application to the estimation of the mean

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Text of the abstract

The most popular extreme value index estimator is the Hill estimator which was introduced in 1975. In this paper, under survival and extreme value theories, we use the estimation of Weissman type of the extreme quantiles for random censorship to present another formula for the mean estimator based on Hill estimator and we construct the bootstrap percentile confidence intervals for this estimator. Finally we apply our results to a real dataset of insurance losses.

Keywords: Hill estimator , Extreme value , Random censoring.
2020 Mathematics Subject Classification: 62G32, 62N02, 62P05.

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# Reflections of Developed Problem Posing Based Active Learning Activities in the Teaching Process: Example of Fractions

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The aim of this study is to develop problem posing based active learning activities for sixth grade students on operations with fractions and to reveal the reflections of these activities in the teaching process. The study group of the research consists of 48 sixth grade students, 23 of whom are in the experimental group and 25 in the control group, studying in a secondary school in Ağrı. While problem posing based active learning activities developed by the researchers were carried out in the experimental group, a teaching program suitable for the learning outcomes was carried out in the control group. Seven problem posing based active learning activities were applied to the students in the experimental group for a total of 21 lesson hours for six weeks. Data were collected with a motivation scale and semi-structured interviews. Independent samples t-test was used to analyze the data obtained from the motivation scale. The interviews with six randomly selected students from the experimental group were audio recorded and the audio recordings were analyzed. Although the mean scores of the experimental group from the motivation scale were not significant [$t(46) = 0.345$, $p > 0.05$], it was higher than the control group. In the interviews, findings were obtained that the students' interest in mathematics increased after problem posing based active learning activities, their anxiety about fractions decreased, they found the activities useful, and they developed the habit of problem posing in and out of the classroom after the activity In the light of the findings, it was concluded that problem posing based active learning activities had positive reflections in the teaching process. This result is in line with the results of the research that mentions the positive contribution of problem posing activities to students' affective characteristics [1,2]. It has been determined that these activities are applicable and can be used in teaching.

Keywords: Problem posing, active learning, motivation..

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# Receiving Student Opinions Within The Scope of Geometry Lessons Taught Using Activities Regarding Different Demonstration-Performance Methods

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The present research was carried out in order to get the opinions of the 7th grade students of the secondary school within the scope of geometry lessons taught by using different demonstration-performance methods in geometry teaching. In the research, the case study method, one of the qualitative research approaches, was used within the scope of taking the views of the students about the geometry lessons taught with the activities using the compass-ruler and origami methods. A semi-structured interview form was applied by the researcher to get students' opinions about the activities made. It can be stated as a result of this study that the activities contributed to the students' understanding of the basics of geometry, their learning with a constructivist approach by assimilating geometry, and the development of their geometry thinking skills.

Keywords:Geometry teaching, Demonstration-Performance Method, Compass-Straightedge, Origam

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# Examining Secondary School 7th Grade Mathematics Activities within the Scope of Harezmian Education Model and Obtaining Students' Opinions

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The present research was carried out in order to examine the activities carried out within the scope of the model for 7th graders and to get the opinions of the students in 2 schools where the Harezmian Education Model (HEM) was applied in the province of Iğdır. In the research, a case study, one of the qualitative research methods, was applied to examine the mathematics activities in the lessons in which HEM was applied. HEM practice lessons were observed by the researcher, a semi-structured interview form was applied to get students' opinions about HEM mathematics applications after the lesson, and the obtained data were interpreted using content analysis method. It can be stated as a result of this study that the majority of students' views on HEM are positive, studies are carried out in accordance with the goal that the model wants to achieve, and it contributes to the students' approach to the problems in their daily lives with mathematical thinking skills by eliminating their prejudices against the mathematics course in which they develop negative attitudes. It is thought that the next studies will be guided by giving place to sample plans, practices and activities related to the process of HEM practice courses in the research.

Keywords:Harezmian Education Model, mathematics education, student views

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# A new primal-dual interior-point algorithm for linear programming

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The propose of this paper is to improve the complexity results of primal-dual interior-point methods for linear optimization (LO) problem. We define a new proximity function for (LO) by a new kernel function wich is a combination of the classic kernel function and a barrier term. We present various proprieties of this new kernel function. Futhermore, we formilate an algorithm for a large-update primal-dual interior-point method (IPM) for (LO). It is shown that the iteration bound for large-update and smal-update primal-dual interior points methods based on this function is a good as the currently best know iteration bounds for these type of methods.We show that the best result of iteration bounds for large-update methods can be achieved, namely $\mathcal{O}(q\sqrt{n}(\log\sqrt{n})^{\frac{q+1}{q}}\log\frac{n}{\epsilon})$ With a special choice of the parameter $q$, the iteration complexity becames

$$\mathcal{O}(\sqrt{n}\log n\log\frac{n}{\epsilon})$$

and for small-update methods is

$$\mathcal{O}(q^{\frac{3}{2}}(\log\sqrt{q})^{\frac{q+1}{q}}\sqrt{n}\log\frac{n}{\epsilon}).$$

This result decreases the gap between the practical behaviour of the large-update algorithms and their theoretical performance, which is an open problem.The primal-dual algorithm is implemented with different choices of the step size.

Numerical results show that the algorithm with practical and dynamic step sizes is more efficient than that with fixed (theoretical) step size.

Keywords: Kernel function, Interior point algorithms, Linear optimization, Complexity bound, Primal-dual methods.

2020 Mathematics Subject Classification : 90C05, 90C51.

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# FULL TEXTS

# Comparative Theoretical and Practical Study Of Some Imaging Algorithm.

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Image restoration is an interesting ill-posed problem. It has a crucial importance in the notion of image processing. We seek to recover an image close to the original image from images disturbed by additive white Gaussian noise. There are many denoising algorithms, in this work we compare the theoretical and numerical results obtained by Tykhonov Regularization, ROF and split Bregman algorithm Anisotropic TV, Isotropic TV in terms of image quality and convergence. Based on the results obtained, we can conclude that the Anisotropic TV denoising and Isotropic TV denoising algorithms work with a direct correlation relationship, the algorithms converge monotonically, Isotropic TV denoising is faster than Anisotropic. Therefore, the results obtained by the ROF algorithm give better high quality denoised image results than the Tykhonov algorithm because the image restored by Tykhonov is too smooth, the edges are eroded. The experimental results show that the methods not only provide better visual resolution, but it can also remove the additive Gaussian noise. We obtain satisfactory results, and we calculate the SNR of each algorithm.

Keywords: Algorithms, Image Restoration, Numerical analysis.
2020 Mathematics Subject Classification: Primary 65D15, Secondary 65K05, 65A05.

## 1 Introduction

The fundamentally erroneous nature of some practical situations is recognized, and it manifests itself in a vast category of problems known as "inverse problems." There are many different sorts of ill posed inverse problems, and they can be found in a variety of fields, such as image processing.

One of the most important aspects of machine learning or computer vision is digital image processing. The restoration of deteriorated photographs is an intriguing subject in digital image processing. During the acquisition of a picture (particularly through photography), it is common for the final image to diverge from the expected image. Debruiting is the converse problem of eliminating noise from an image; the result would be poor if noise was not removed from the image. Brièvement, Noise is parasitic information that is added to the scene at random. Noise is diverse in nature and origin, thus it can be simulated in a variety of ways. There are various types of noise; the Gaussian additive noise is the case study in this article: $f = u + v$ ,with $f$ representing the observed noisy image, $u$ representing the original image, and $v$ representing Gaussian random variation to zero mean. In a predetermined density function, Gaussian noise is also known as normal noise. It's a popular way to add noise to an image. This noise can be generated in the image at random and individually, as defined by the following.

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\overline{z})^{2/2\sigma^2}} \tag{1}$$

where $z$ stands for intensity, $\overline{z}$ represents the mean value of $z$, and $\sigma$ stands for standard deviation. As seen in Fig. 1, this function can be visualized.

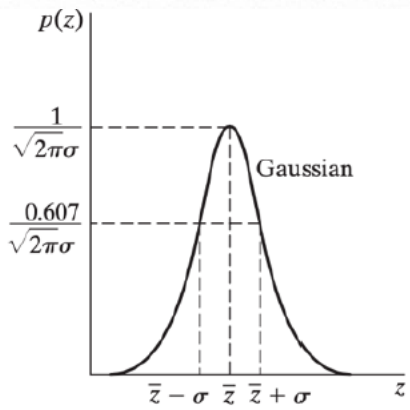


Figure 1: Gaussian Noise

De-escalation techniques are required to restore the image to a higher visual quality. This article will address the investigation of various image restoration models with Gaussian noises.

## 2 Comparative Studies of Some Algorithms In Image Processing.

Image representation is a technique for repairing or degrading an image while lowering the amount of noise it contains.

We will discuss some image restore models that will be used in the experiment in the sections below.

## 2.1 Tychonov regularization

One commance by the first model which most encient model of Tychonov Regularization . This model uses the functional space of Sobolev $H_0^1(\Omega)$ which is a separable Hilbert space.

Space $H_0^1(\Omega)$ is equipped with the scalar product induced by $H^1$ :

$$\|u\|_{H^1} = \|u\|_{L^2} + \left\|u'\right\|_{L^2} \tag{2}$$

Tychonov regularization this is a very conventional but too basic regularization process for image processing. We try to reconstruct or restore the image $u$, if we assume that the additive noise $v$ is gaussian and $f$ is the observed image.

Let $V = H_0^1(\Omega)$ et $H = L^2(\Omega)$, We consider the original minimisation problem (adjustment to the data):

$$(p) \qquad \min_{u\in V} \|u - f\|_H^2 \tag{3}$$

where $f : \Omega \subset \mathbb{R}^N \longrightarrow R$ is the observed image and the following regularized problem: for any $\alpha > 0$

$$(p_\alpha) \qquad \min_{u\in V} \left\{\|u - f\|_H^2 + \alpha \|\nabla u\|_H^2\right\} \tag{4}$$

We only want to adjust u to the data f , but we also impose that the gradient is "quite small" (it depends on the parameter ). An image with a small gradient is "smoothed". The edges are eroded and the restore will give a blurred image.

**Theorem 2.1** *Assume that $(P)$ requires at least one answer $\breve{u}$. The problem $(P_\alpha)$ requires a one-of-a-kind solution $u_\alpha$. When $\alpha \to 0$, one can extract a sub-suite from the family $(u_\alpha)$ that converges (possibly) in $V$ to a solution $u^*$ of $(P)$.*

The traditional phrase for image restoration, $L(u) = \|\nabla u\|_2^2$ (Tychonov regularization), is incompatible with the problem of image restoration: the image restored $u$ is much too lissée (in particular, the edges are eroded). Consider the total variation, that is, take $L(u) = \int |Du|$. This is a far more effective approach. This results in a functional minimization in a specific Banach space, which is well suited to the problem of variation-based function spaces.

## 2.2 Model Rudin -Osher-Fatemi (ROF)

Rudin Osher and Fatemi $(ROF)$ , proposed the first image resetting model from a given noisy image having additive noise using regularization $(TV)$ which defines by :

$$TV(u(x,y)) =_\Omega |\nabla u(x,u)|\, dxdy \quad avec\, |\nabla u| = \sqrt{u_x^2 + u_y^2} \tag{5}$$

To remove noise from digital images, the method of regularization of total variation $(TV)$ of image processing is used. It is the only regularization that is used to preserve the image's borders and eliminate the image's frequently bruyant components. $(TV)$ is a technique created by Rudin- Osher and Fatemi to solve the problem of image degradation. Since then, it has been applied to a variety of other image problems.

In (3), a model has been proposed by Rudin-Osher-Fatemi. In which the image is divided into two parts: $f = u + v$, where $v$ is the noise and $u$ is the "regular" half, $u$ is an unknown image, and $f$ is a bruyant measure typical at the start of a clean image and is an agreement parameter. So, using the $u + v$ formula with $u \in BV(\Omega)$ et $v \in L^2(\Omega)$, we'll look for a solution to the problem and only apply the regularization to the "bruit" section. If $f \in L^2$ is correct, then the minimizer $u$ exists, is unique, and stable in $L^2$, and the $ROF$ problem is well-posed.

$ROF$ proposed the following minimization problem:

$(\mathrm{P}_{ROF})\qquad \inf_u \left\{ J(u) + \frac{1}{2\lambda}\|v\|_2^2 / u \in BV(\Omega),\ v \in L^2(\Omega), f = u + v \right\}$

This results in a $[BV(\Omega), L^2(\Omega)]$ decomposition of the image $f$.

$J(u)$ denotes the total variance of $u$ and $\lambda > 0$.

$$J(u) = \sup\left\{ \int_\Omega u(x)div(\varphi(x))dx : \varphi \in C_c^1\left(\Omega, \mathbb{R}^2\right), \|\varphi\|_\infty \leq 1 \right\} \tag{6}$$

And $BV$, is Borne Variable Function Space, as defined by:

$$BV(\Omega) = \left\{ u \in L^1(\Omega), J(u) < +\infty \right\} \tag{7}$$

Here $J(u)$ denotes the $TV$ of $u$ and $\lambda > 0$ is a weight parameter. $BV$ is a space of bounded variation functions ( the space of all images at $TV$ ).

**Theorem 2.2** *(3), The problem $(P_{ROF})$ requires a single solution, which is provided by*

$$u = f - \lambda \Pi_{\lambda K}(f) \tag{8}$$

*where $\Pi$ is the orthogonal projector on $\lambda K$ (dilatation of $K$ by$\lambda$ ), and $K$ is the overall closure in $L^2$:*

$$K := \left\{ div(\varphi)/\varphi \in C_c^1\left(\Omega, \mathbb{R}^2\right), \|\varphi\|_\infty \leq 1 \right\} \tag{9}$$

## 2.3 Bregman Algorithm

Bregman's iterative technique was first introduced and studied in the field of image processing by Osher et al (10). Osher, Burger, Goldfarb, Xu, and Yin proposed the iterative Bregman algorithm as an effective algorithm for solving optimization problems (6). Their main ideas were to first transform a constraint optimization problem into a constraint-free problem using the Bregman distance. This problem-solving algorithm is as follows:

$$\min_u \left\{ z(u) + H(u, f) \right\} \tag{10}$$

such that $z : X \longrightarrow \mathbb{R}$ , $H : X \longrightarrow \mathbb{R}$ are non-negative convex functions of $u \in X$, and $H(u, f)$ is a smooth non-negative convex function in relation to $u$ for a given $f$, and $X$ is a closed convex set.

Bregman's iterative algorithm is defined as follows by Osher, Burger, Goldfarb, Xu, and Yin: Bregman Iterative Algorithm

- Initialize: : $k = 0$, $u_0 = 0$, $p_0 = 0$.
- while "$u^k$ not converge "$u^{k+1} \leftarrow \arg\min D_z^{P^k}(u, u^k) + H(u)$
- $p^{k+1} \leftarrow p^k - \nabla H(u^{k+1}) \in \partial z(u^{k+1})$
- $k \leftarrow k + 1$
- End while

### 2.3.1 The Convergence Theorem

For the first time in (10), this variant of Bregman was presented for TV-based image rendering. Other features of this iterative Bregman scheme, as well as a convergence analysis, have been proven in detail in ((10), (11), and (5). $u^1 = u\min(z(u) + H(u))$ is the first iteration of this method.

The residual term must be minimal to solve the initial problem; once the residual term converges, the Bregman iterative algorithm continues. Because of its excellent convergence features, the Bregman iterative algorithm has been applied to a variety of issues, including badly posed problems and picture dissection. Among these qualities are: with noisy data, we can achieve convergence to the original image we're seeking to recover, as well as convergence in terms of distance from Bregman to the original image, and a monotonous decline in the residual term. We have $z(u) = \|u\|_{BV}$ where $\|\nabla u\|_1$ and $H(u) = \frac{1}{2}\|u - f\|_2^2$ learned a lot about image redaction during our research.

## 2.4 Algorithm of Bregman Splitting

Goldstein and Osher first proposed the Split Bregman algorithm in (9) to handle more general form optimization problems:

$$\min_{u \in X} (H(u) + \|\Phi(u)\|_1), \tag{11}$$

where $X$ is a closed convex set, and $\Phi : X \longrightarrow \mathbb{R}$, $H : X \longrightarrow \mathbb{R}$ are convex functions. This problem is the same as the stress minimization problem below:

$$\min_{u \in X, d \in \mathbb{R}} (H(u) + \|d)\|_1) \textit{ such that } d = \Phi(u). \tag{12}$$

Goldstein and Osher introduced the Split Bregman algorithm, which was written as follows: Algorithm of Bregman Splitting

- Initialization :$k = 0, u^0 = 0, b^0 = 0$
- As long as $||u^k - u^{k-1}|| > tol$ do
- $u^{k+1} = \min_u \ H(u) + \frac{\lambda}{2} \left\| d^k - \Phi(u) - b^k \right\|_2^2$
- $d^{k+1} = \min_d \ |d| + \frac{\lambda}{2} \left\| d - \Phi(u^{k+1}) - b^k \right\|_2^2$
- $b^{k+1} = b^k + (\Phi(u^{k+1}) - d^{k+1})$
- $k = k + 1$
- End as long as

The Split Bregman algorithm is used to solve some of the most common form optimization problems:

$$\min_{u \in X} (z(u) + \frac{1}{2} \|u - f\|_2^2). \tag{13}$$

Anisotropic and isotropic TV disconnection problems are solved using the Split Bregman method.

### 2.4.1 Denoising Anisotropic TV

The problem of anisotropic TV Denoising is considered in (5)

$$\min_u (\left\| \frac{\partial u}{\partial x} \right\|_1 + \left\| \frac{\partial u}{\partial y} \right\|_1 + \frac{\mu}{2} \|u - f\|_2^2), \qquad (\mathrm{P}_1)$$

where $f$ is the noisy image, $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ will be noted by $u_y$ and $u_x$ respectively. The problem is solved using a constraint equivalent to a problem $(P1)$ We answer the problem $(P2)$ as follows:

$$\begin{cases} \min_u (\|d_x\|_1 + \|d_y\|_1 + \frac{\mu}{2} \|u - f\|_2^2) \\ with \quad d_x = u_x, d_y = u_y. \end{cases} \qquad (\mathrm{P}_2)$$

The Split Bregman algorithm can be used to tackle this last problem.

$$\min_{u, d_x, d_y} (\|d_x\|_1 + \|d_y\|_1 + \frac{\mu}{2} \|u - f\|_2^2 + \frac{\lambda}{2} \|d_x - u_x\|_2^2 + \frac{\lambda}{2} \|d_y - u_y\|_2^2). \qquad (\mathrm{P}_3)$$

We establish

$$shrink(x, a) = \begin{cases} x - a & if x > a, \\ x + a & if x < a, \\ 0 & else. \end{cases} \tag{14}$$

The Gauss Seidel function is also useful

$$G_{i,j}^k = \frac{\lambda}{\mu+4\lambda}(u_{i+1,j}^k + u_{i-1,j}^k + u_{i,j+1}^k + u_{i,j-1}^k + d_{x,i-1,j}^k + d_{x,i,j}^k$$
$$+d_{y,i,j-1}^k + d_{y,i,j}^k + b_{x,i-1,j}^k + b_{x,i,j}^k + b_{y,i,j-1}^k + b_{y,i,j}^k) + \frac{\mu}{\mu+4\lambda} f_{i,j}. \quad (15)$$

The Split Bregman algorithm of denoising Anisotropic TV Sound Effects:

- Initialization: $:k = 0, u^0 = 0, b^0 = 0$
- As long as $||u^k - u^{k-1}|| > tol$ do
- $u^{k+1} = G^k$ où $G$ is the Gauss-Seidel function
- $d_x^{k+1} = shrink\ (\nabla_x u^{k+1} + b_x^k, \frac{1}{\lambda})$
- $d_y^{k+1} = shrink\ (\nabla_y u^{k+1} + b_y^k, \frac{1}{\lambda})$
- $b_x^{k+1} = b_x^k(\nabla_x u^{k+1} - d_x^{k+1})$
- $b_y^{k+1} = b_y^k(\nabla_y u^{k+1} - d_y^{k+1})$
- $k = k + 1$
- End as long as

### 2.4.2 Denoising Isotropic TV

The problem of Isotropic TV denoising is considered in (5)

$\min_u \|\nabla u\|_2 + \frac{\mu}{2} \|u - f\|_2^2. \qquad (\mathrm{P}'_1)$

The problem is solved using a constraint equivalent to a problem $(P'_2)$

$$\begin{cases} \min_u(\|(d_x, d_y)\|_2 + \frac{\mu}{2} \|u - f\|_2^2) \\ with \quad d_x = u_x, d_y = u_y. \end{cases} \qquad (\mathrm{P}'_2)$$

To solve the problem $(P'_2)$, we solve the following problem without constraint:

$$\min_{u,d_x,d_y}(\|(d_x, d_y)\|_2 + \frac{\mu}{2} \|u - f\|_2^2 + \frac{\lambda}{2} \|d_x - u_x\|_2^2 + \frac{\lambda}{2} \|d_y - u_y\|_2^2). \qquad (\mathrm{P}'_3)$$

The Split Bregman algorithm can be used to tackle this last difficulty.

We give the following definition: $\mathrm{s}^k := \sqrt{|u_x^k - b_x^k|^2 + \left|u^k - b_y^k\right|^2}$.

The Split Bregman algorithm of denoising Isotropic TV Sound Effects:

- Initialization: $k = 0, u^0 = 0, b^0 = 0$
- As long as $\left\|u^k - u^{k+1}\right\| > tol$ do
- $u^{k+1} = G^k$ où $G$ is the Gauss-Seidel function
- $d_x^{k+1} = \frac{s^k \lambda(u_x^k + b_x^k)}{s^k \lambda + 1}$
- $d_y^{k+1} = \frac{s^k \lambda(u_y^k + b_y^k)}{s^k \lambda + 1}$
- $b_x^{k+1} = b_x^k + (u_x^{k+1} - d_x^{k+1})$
- $b_y^{k+1} = b_y^k + (u_y^{k+1} - b_y^{k+1})$
- $k = k + 1$
- End as long as

## 3 Numerical Results

Let $X$ be a representation of an " University" original image. We defined our noisy image $f$ using the Matlab command $f = imnoise(X,' gaussian', segma)$ where segma is a variant of the Gaussian noise level. For our experiments, We used the values $\mu = 0.1$, $\lambda = 0.2$ and the tolerance $Tol = 10^{-3}$ . Also the

figures$(2; 4)$ illustrate that the original image and noisy image for different segma values: 0.08 *and* 0.35 and the

figures $(3; 5)$ illustrate that the different algorithms applied to restoration image using: Tychonov regularization ,the modele ROF , Isotropic TV denoising algorithm and Anisotropic TV denoising algorithm for "University" image for different segma values. In Tables 1 and 2, we show the results for the Anisotropic TV and Isotropic TV denoising algorithms for image "University", and different values segma . The relative error is measured by : $||u-X||_2^2$ ,we also note number of iteration , relative error and time. In Tables 3 , we show The different values of SNR of denosed image by: Tychonov regularization, the model ROF, the Anisotropic TV and Isotropic TV denoising algorithms.

| Segma | SNR_TV_AS | Number of iteration | Relative Error | Time(s) |
|---|---|---|---|---|
| 0.08 | 13.1821 | 7 | 0.14374 | 45.056700 |
| 0.15 | 10.3202 | 6 | 0.265516 | 55.542690 |
| 0.25 | 6.9137 | 6 | 0.435046 | 28.271569 |
| 0.35 | 4.4097 | 5 | 0.597903 | 23.069885 |
| 0.5 | 1.8064 | 5 | 0.819539 | 24.158445 |

Table 1: Results for Image university Anisotropic TV denoising algorithm


Figure 2: Original image and noisy image for segma 0.08

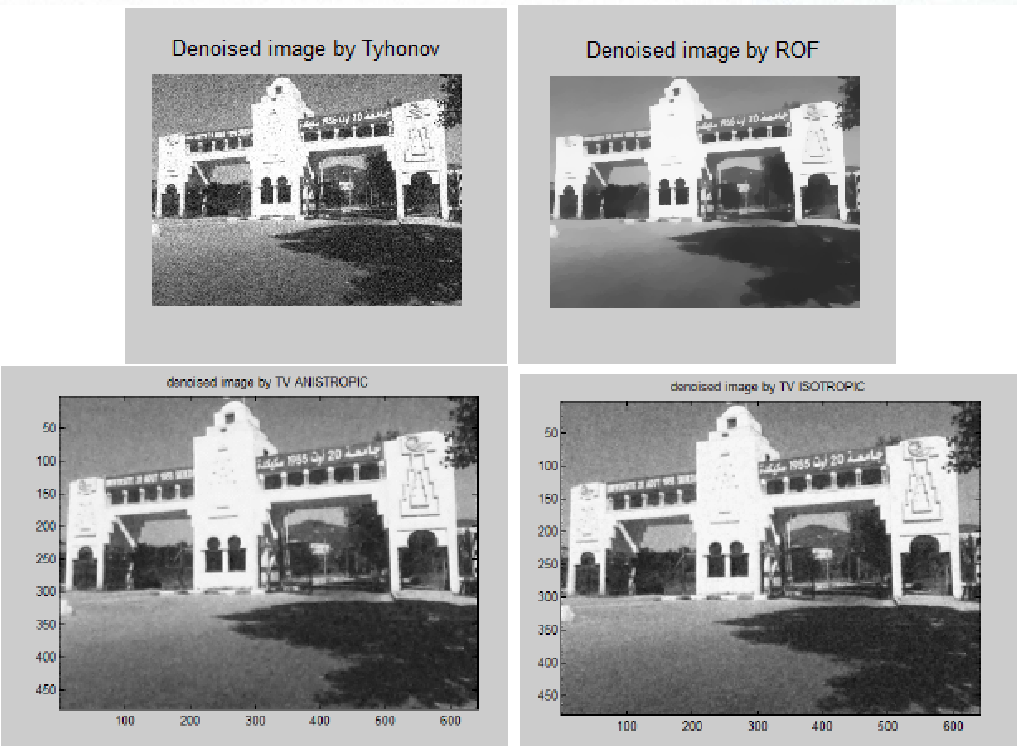


Figure 3: The results of denosed images for segma 0.08

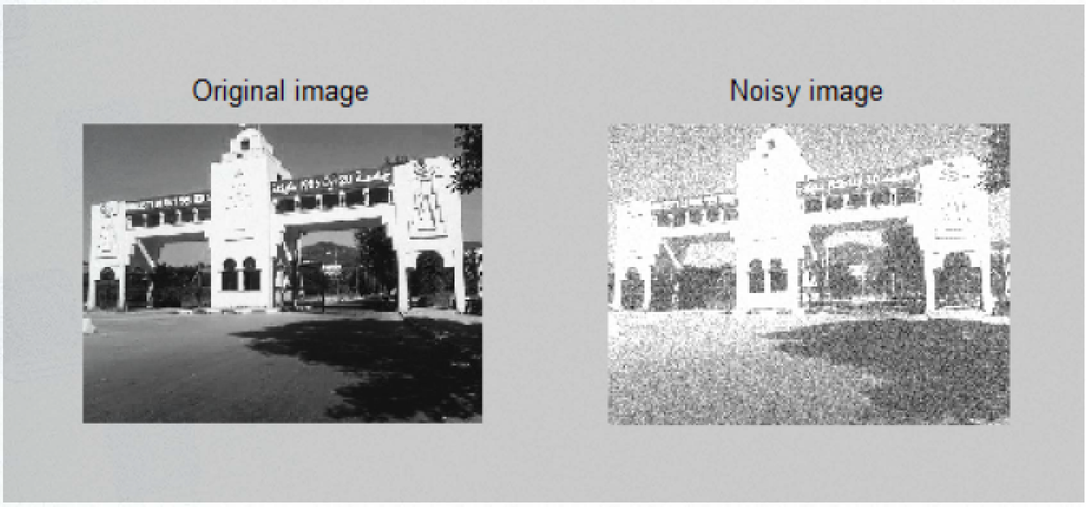


Figure 4: Original image and noisy image for segma 0 : 35

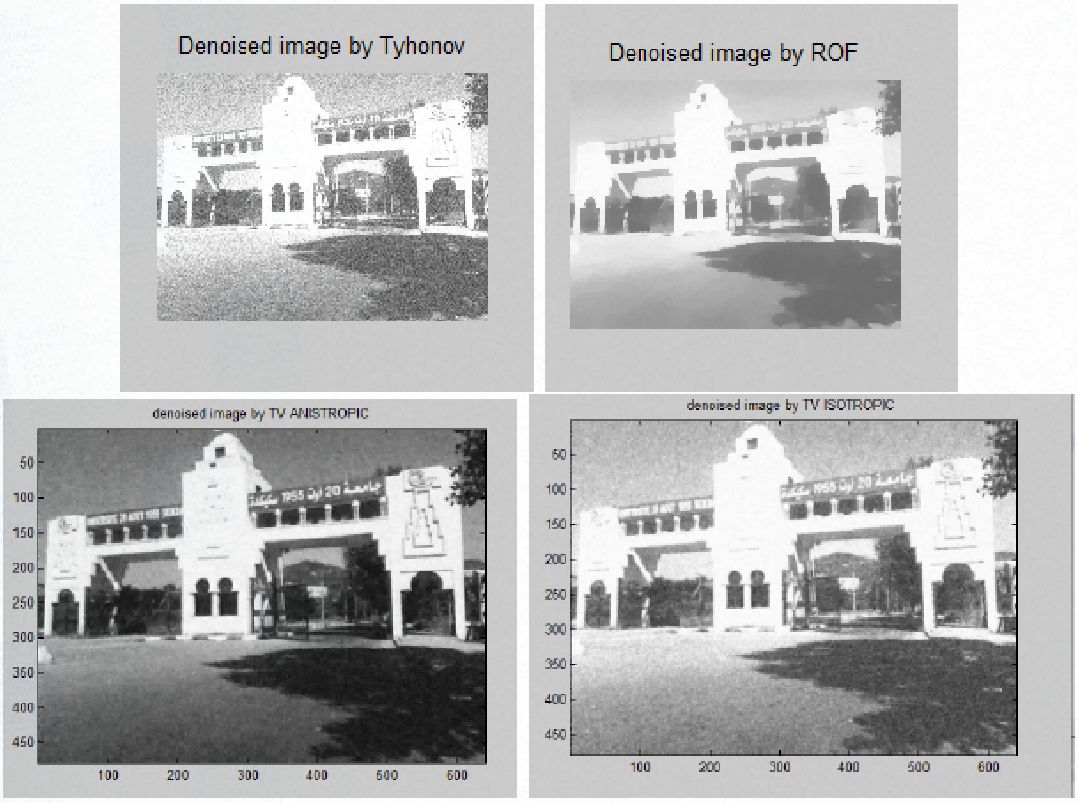


Figure 5: The results of denosed images for segma 0.35

| Segma | SNR_TV_IS | Number of iteration | Relative Error | Time(s) |
|---|---|---|---|---|
| 0.08 | 13.1742 | 7 | 0.143628 | 31.280807 |
| 0.15 | 10.3192 | 6 | 0.265229 | 25.489589 |
| 0.25 | 6.9151 | 6 | 0.434996 | 26.993944 |
| 0.35 | 4.4145 | 5 | 0.597577 | 21.442199 |
| 0.5 | 1.8096 | 4 | 0.819026 | 19.234112 |

Table 2: Results for Image university Isotropic TV denoising algorithm

| Segma | SNR_TV_AS | SNR_TV_IS | SNR ROF | SNR Tykhonov |
|---|---|---|---|---|
| 0.08 | 13.1742 | 13.1742 | 2.2494 | 2.2494 |
| 0.15 | 10.3192 | 10.3192 | 1.0372 | 1.0372 |
| 0.25 | 6.9151 | 6.9151 | 0.4657 | 0.4657 |
| 0.35 | 4.4145 | 4.4145 | 0.3858 | 0.3858 |
| 0.5 | 1.8096 | 1.8096 | 0.3813 | 0.3813 |

Table 3: The different values SNR of denoising image b Tykhonov; ROF; the Anisotropic TV and the Isotropic TV .

# 4 Conclusion

Based on the above results, we can conclude that the algorithms of $TV$ Anisotrope and $TV$ Isotropen work with a direct relationship, the algorithm converges monotonously, the $TV$ Isotropic noise is faster than Anisotrope. Consequently, the results obtained by the modele the $ROF$ of denoising image shows that the regularization term has more in influence on the energy and therefore on the position because they used the Borne Variable Function Space $BV$ . The diferent values of $SNR$ of denosed image by: Tychonov regularization, model $ROF$, are almost close and the $TV$ Anisotropic and $TV$ Isotropic denoising algorithms also the expressed results confirm that the methods not only ofers a better visual response, but that it can also remove additive wafer noise.

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# Some completely monotonicity properties and related inequalities involving $k$-trigamma and $k$-tetragamma functions

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**Abstract**

A function $f$ is said to be completely monotonic on an interval $I$ if $f$ has derivatives of all order on $I$ and $0 \leq (-1)^n f^{(n)}(x) < \infty$ for $\forall x \in I$ and $n \geq 0$. Also, BernsteinWidder Theorem in [1, Theorem 12a, p. 160] states that a function $f(x)$ on $[0, \infty)$ is completely monotonic if and only if there exists a bounded and non-decreasing function $\alpha(t)$ such that $f(x) = \int_0^\infty e^{-xt} d\alpha(t)$ converges for $\forall x \in [0, \infty)$. This says that a completely monotonic function $f(x)$ on $[0, \infty)$ is the Laplace transform of the measure $\alpha(t)$. In this work, we motivate by the previous information, development in $k$-special functions such as properties on $k$-gamma function (see [2]), integral representations on $k$-digamma function in [3] and inequalities obtained by Yıldırım in [4]. At first, we show the complete monotonicity of the function defined by $[\psi_k'(x)]^2 + \frac{1}{k}\psi_k''(x)$ and then obtain double sided inequalities on the function for all positive real values of $x$ and $k$. The results in this work are $k$-generalizations of the classical ones in [5, 6, 7].

Keywords: Complete monotonicity, Inequalities, $k$-digamma function.
2020 Mathematics Subject Classification: 26A48, 33B99, 26D07.

## 1 Preliminaries and Motivation

Many researchers interest in special functions and obtain inequalities, monotonicity properties or generalizations on these functions. Such as Díaz and Pariguan introduced $k$-Pochhammer symbol in order to define $k$-gamma function $\Gamma_k$ as the following limit expression:

**Definition 1.1.** *[2] Let $x \in \mathbb{C}$, $k \in \mathbb{R}$ and $n \in \mathbb{N}^+$, the Pochhammer $k$-symbol is given as*

$$(x)_{n,k} = x(x+k)(x+2k)\dots(x+(n-1)k).$$

**Definition 1.2.** *[2] For $k > 0$, the $k$-gamma function $\Gamma_k$ is given by*

$$\Gamma_k(x) = \lim_{n\to\infty} \frac{n!k^n(nk)^{\frac{n}{k}-1}}{(x)_{n,k}}, \quad x \in \mathbb{C} \setminus k\mathbb{Z}^-.$$

Dí az and Pariguan also obtained its integral and infinite product representations by

$$\Gamma_k(x) = \int_0^\infty u^{x-1} e^{-\frac{u^k}{k}} du \tag{1.1}$$

$$\frac{1}{\Gamma_k(x)} = x k^{-\frac{x}{k}} e^{\frac{x}{k}\gamma} \prod_{n=1}^{\infty} \left( \left(1 + \frac{x}{nk}\right) e^{-\frac{x}{nk}} \right) \tag{1.2}$$

for $x \in \mathbb{C}$, $Re(x) > 0$. They proved some properties on $k$-gamma function such as

$$\Gamma_k(x+k) = x\Gamma_k(x) \tag{1.3}$$

$$\Gamma_k(x) = k^{\frac{x}{k}-1}\Gamma\left(\frac{x}{k}\right). \tag{1.4}$$

Authors in [3] found several integral representations of $k$-digamma function. One of them is defined by

$$\psi_k(x) = \frac{\ln k}{k} + \frac{\gamma}{k} + \int_0^1 \frac{u^{k-1} - u^{x-1}}{1-u^k} du \tag{1.5}$$

for $x,\ k > 0$. By substituting $e^{-t} = u$, $-e^{-t}dt = du$ and $t : \infty \to 0$ in the integral representation of $k$-digamma function (1.5) and then differentiating the equation one can obtain the following results:

**Lemma 1.3.** *[8] For all positive real values of $x$ and $k$ and positive integer $n$, $k$-digamma and $k$-polygamma functions can be defined as the following integrals:*

$$\psi_k(x) = \frac{\ln k - \gamma}{k} + \int_0^\infty \frac{e^{kt} - e^{-xt}}{1-e^{-kt}} dt \tag{1.6}$$

$$\psi_k^{(n)}(x) = (-1)^{n+1} \int_0^\infty \frac{t^n}{1-e^{-kt}} e^{-xt} dt. \tag{1.7}$$

Taking logarithmic derivative of the equation (1.4) leads us to the recurrence formula for $k$-digamma function by

$$\psi_k(x+k) = \frac{1}{x} + \psi_k(x) \tag{1.8}$$

and for first and second derivatives of the equation (1.8) that are called $k$-trigamma and $k$-tetragamma functions respectively, we get

$$\psi_k'(x+k) = \psi_k'(x) - \frac{1}{x^2} \tag{1.9}$$

$$\psi_k''(x+k) = \psi_k''(x) + \frac{2}{x^3} \tag{1.10}$$

respectively for $x,\ k > 0$.

Yıldırım in [4] uses Binet's first formula for the logarithms of $k$-gamma function $\ln \Gamma_k(x)$ and complete monotonicity properties on $k$-digamma function and its derivatives to obtain following inequalities:

**Corollary 1.4.** *[4] The following inequalities*

$$\frac{\ln x}{k} - \frac{1}{2x} - \frac{k}{12x^2} < \psi_k(x) < \frac{\ln x}{k} - \frac{1}{2x}, \tag{1.11}$$

$$\frac{1}{kx} + \frac{1}{2x^2} + \frac{k}{6x^3} - \frac{k^3}{30x^5} < \psi_k'(x) < \frac{1}{kx} + \frac{1}{2x^2} + \frac{k}{6x^3} \tag{1.12}$$

*and*

$$-\frac{1}{kx^2} - \frac{1}{x^3} - \frac{k}{2x^4} < \psi_k''(x) < -\frac{1}{kx^2} - \frac{1}{x^3} \tag{1.13}$$

*are valid for all $x, k > 0$.*

By using previous inequalities (1.12) and (1.13) and the recurrence formula (1.9), we can easily derive the following result:

**Remark 1.5.** *The following double sided inequalities*

$$\left[\frac{1}{x^2}+\frac{1}{k(x+k)}+\frac{1}{2(x+k)^2}+\frac{k}{6(x+k)^3}-\frac{k}{30(x+k)^5}\right]^2-\frac{1}{k}\left(\frac{1}{kx^2}+\frac{1}{x^3}+\frac{k}{2x^4}\right)$$
$$<[\psi_k'(x)]^2+\frac{1}{k}\psi_k''(x)\left[\frac{1}{x^2}+\frac{1}{k(x+k)}+\frac{1}{2(x+k)^2}+\frac{k}{6(x+k)^3}\right]^2-\frac{1}{k}\left(\frac{1}{kx^2}+\frac{1}{x^3}\right)$$

*i.e.*

$$\frac{p_k(x)}{900x^4(x+k)^{10}}<[\psi_k'(x)]^2+\frac{1}{k}\psi_k''(x)<\frac{q_k(x)}{36x^4(x+k)^6} \tag{1.14}$$

*are valid, where the functions $p_k$ and $q_k$ are defined by*

$$p_k(x)=75x^{10}+900kx^9+4840k^2x^8+15370k^3x^7+31865k^4x^6+45050k^5x^5$$
$$+44101k^6x^4+29700k^7x^3+13290k^8x^2+3600k^9x+450k^{10}. \tag{1.15}$$

*and*

$$q_k(x)=21x^6+132kx^5+352k^2x^4+504k^3x^3+408k^4x^2+180k^5x+36k^6 \tag{1.16}$$

*respectively.*

We want to note that the left side of the inequality (1.14) is a $k$-generalization that is obtained by Alzer in [5, (4,39), p.208]. The function defined by $\left[\psi_k'(x)\right]^2+\frac{1}{k}\psi_k''(x)$ is interested by many researchers. In [8] authors obtain the following results:

**Theorem 1.6.** *[8] The functions*

$$P(x)=\left[\psi_k'(x)\right]^2+\frac{1}{k}\psi_k''(x)-\frac{x^2+12k^2}{12x^4(x+k)^2} \tag{1.17}$$

*and*

$$Q(x)=\frac{x+12k}{12x^4(x+k)}-\left[\psi_k'(x)\right]^2-\frac{1}{k}\psi_k''(x) \tag{1.18}$$

*are completely monotonic for all positive real values of $x$ and $k$. As an immediate consequence, the following double sided inequalities*

$$\frac{x^2+12k^2}{12x^4(x+k)^2}<\left[\psi_k'(x)\right]^2+\frac{1}{k}\psi_k''(x)<\frac{x+12k}{12x^4(x+k)} \tag{1.19}$$

*are valid.*

Motivated by previous results and classical generalizations, we firstly show completely monotonicity properties on the function related to $[\psi_k'(x)]^2+\frac{1}{k}\psi_k''(x)$ for all real values $x$ and $k$.

# 2 Main Results

At first, we need the following property:

**Lemma 2.1.** *We have*

$$\frac{1}{x^{r/k}}=\frac{k^{r/k-1}}{\Gamma_k(r)}\int_0^\infty t^{r/k-1}e^{-xt}dt \tag{2.1}$$

*for $\forall x,\ k,\ r\in\mathbb{R}^+$.*

*Proof.* By substituting $u^k/k = xt$, $u^{k-1}du = xdt$ in the integral representation of $k$-gamma function (1.1), one can get desired result. We want to note that for $n \in \mathbb{Z}^+$, the equation (2.1) becomes

$$\frac{1}{x^n} = \frac{1}{(n-1)!}\int_0^\infty t^{n-1}e^{-xt}dt \tag{2.2}$$

by using the equation $\Gamma_k(nk) = (n-1)!k^{n-1}$ □

Now we will ready to show the complete monotonicity on the function related to $[\psi_k'(x)]^2 + \frac{1}{k}\psi_k''(x)$:

**Theorem 2.2.** *The functions*

$$F(x) = [\psi_k'(x)]^2 + \frac{1}{k}\psi_k''(x) - \frac{p_k(x)}{900x^4(x+k)^{10}} \tag{2.3}$$

*and*

$$G(x) = \frac{q_k(x)}{36x^4(x+k)^6} - [\psi_k'(x)]^2 - \frac{1}{k}\psi_k''(x) \tag{2.4}$$

*are completely monotonic for all positive values of $x$ and $k$, where the functions $p_k$ and $q_k(x)$ are defined by (1.15) and (1.16) respectively.*

*Proof.* Using recurrence formula (1.9) and (1.10) yields that

$$\begin{aligned}
F(x) - F(x+k) &= \left[\psi_k'(x) - \psi_k'(x+k)\right]\left[\psi_k'(x) + \psi_k'(x+k)\right] + \frac{1}{k}\left[\psi_k''(x) - \psi_k''(x+k)\right] \\
&- \left[\frac{p_k(x)}{900x^4(x+k)^{10}} - \frac{p_k(x+k)}{900(x+k)^4(x+2k)^{10}}\right] \\
&= \frac{1}{x^2}\left[2\psi_k'(x) - \frac{1}{x^2}\right] - \frac{2}{kx^3} - \left[\frac{p_k(x)}{900x^4(x+k)^{10}} - \frac{p_k(x+k)}{900(x+k)^4(x+2k)^{10}}\right] \\
&= \frac{2}{x^2}\left[\psi_k'(x) - \frac{1}{kx} - \frac{3}{4x^2} + \frac{28}{5k(x+k)} - \frac{251}{120(x+k)^2} - \frac{7k}{6(x+k)^3} + \frac{13k^2}{90(x+k)^4}\right. \\
&- \frac{13k^3}{180(x+k)^5} - \frac{k^4}{120(x+k)^6} + \frac{k^5}{180(x+k)^7} + \frac{k^6}{200(x+k)^8} + \frac{k^7}{900(x+k)^9} - \frac{k^8}{1800(x+k)^{10}} \\
&+ \frac{51}{10k(x+2k)} + \frac{331}{120(x+2k)^2} + \frac{17k}{12(x+2k)^3} + \frac{49k^2}{72(x+2k)^4} + \frac{47k^3}{180(x+2k)^5} \\
&\left.- \frac{k^4}{60(x+2k)^6} - \frac{2k^5}{45(x+2k)^7} - \frac{13k^6}{600(x+2k)^8} - \frac{k^7}{450(x+2k)^9} + \frac{k^8}{450(x+2k)^{10}}\right] \\
&= \frac{2}{x^2}f(x).
\end{aligned}$$

Using the equation (2.2) and integral representation (1.7) leads us to

$$\begin{aligned}
f(x) = \int_0^\infty &\left(\frac{t}{1-e^{-t}} - \frac{1}{2k} - \frac{3t}{4} + \frac{28}{5}e^{-kt} + \frac{251}{120}te^{-kt} - \frac{7k}{12}t^2e^{-kt} + \frac{13k^2}{540}t^3e^{-kt}\right. \\
&- \frac{13k^3}{4320}t^4e^{-kt} - \frac{k^4}{14400}t^5e^{-kt} + \frac{k^5}{129600}t^6e^{-kt} + \frac{k^6}{100800}t^7e^{-kt} + \frac{k^7}{3628800}t^8e^{-kt} \\
&- \frac{k^8}{65318400}t^9e^{-kt} + \frac{51}{10k}e^{-2kt} + \frac{331}{120}te^{-2kt} + \frac{17k}{24}t^2e^{-2kt} + \frac{49k^2}{72.3!}t^3e^{-2kt} \\
&+ \frac{47k^3}{180.4!}t^4e^{-2kt} - \frac{k^4}{60.5!}t^5e^{-2kt} - \frac{2k^5}{45.6!}t^6e^{-2kt} - \frac{13k^6}{600.7!}t^7e^{-2kt} - \frac{k^7}{450.8!}t^8e^{-2kt} \\
&\left.+ \frac{k^8}{450.9!}t^9e^{-2kt}\right)e^{-xt}dt.
\end{aligned}$$

By using similar technique in [6], we can show that the function $f$ is completely monotonic on $(0,\infty)$ and since $\frac{2}{x^2}$ is also completely monotonic and the product of two completely monotonic functions is completely monotonic on the same interval, we conclude that the function $F(x)-F(x+k)$ is completely monotonic for all $x,k>0$, i.e.

$$(-1)^n F^{(n)}(x) - (-1)^n F^{(n)}(x+k) \geq 0.$$

By mathematical induction, the proof is completed. The complete monotonicity property of the function $G(x)$ can be obtained similarly.
We want to note that the desired result can also be obtained by using the relation $\psi_k'(x) = \frac{1}{k^2}\psi'\left(\frac{x}{k}\right)$. The difference $F(x)-F(x+k)$ becomes

$$\begin{aligned}
F(x)-F(x+k) = \frac{2}{x^2}\Bigg[&\frac{1}{k^2}\psi'(x) - \frac{1}{k^2(x/k)} - \frac{3}{4k^2(x/k)^2} + \frac{28}{5k^2(x/k+1)} - \frac{251}{120k^2(x/k+1)^2} \\
&- \frac{7}{6k^2(x/k+1)^3} + \frac{13}{90k^2(x/k+1)^4} - \frac{13}{180k^2(x/k+1)^5} - \frac{1}{120k^2(x/k+1)^6} + \frac{1}{180k^2(x/k+1)^7} \\
&+ \frac{1}{200k^2(x/k+1)^8} + \frac{1}{900k^2(x/k+1)^9} - \frac{1}{1800k^2(x/k+1)^{10}} + \frac{51}{10k^2(x/k+2)} + \frac{331}{120k^2(x/k+2)^2} \\
&+ \frac{17}{12k^2(x/k+2)^3} + \frac{49}{72k^2(x/k+2)^4} + \frac{47}{180k^2(x/k+2)^5} - \frac{1}{60k^2(x/k+2)^6} - \frac{2}{45k^2(x/k+2)^7} \\
&- \frac{13}{600k^2(x/k+2)^8} - \frac{1}{450k^2(x/k+2)^9} + \frac{1}{450k^2(x/k+2)^{10}}\Bigg]
\end{aligned}$$

$$F(x)-F(x+k) = \frac{2}{(kx)^2} H\left(\frac{x}{k}\right).$$

Since authors in [6] show that the function $H$ is completely monotonic on $(0,\infty)$. Therefore the function $F(x)$ is also completely monotonic for all $x,k>0$ □

As an immediate consequence, we can obtain the following result:

**Corollary 2.3.** *The function*

$$S(x) = \psi_k'(x) - \frac{s(x)}{1800kx^2(x+k)^{10}(x+2k)^{10}} \tag{2.5}$$

*is completely monotonic for all positive real values of $x$ and $k$, where*

$$\begin{aligned}
s(x) = {}& 1382400k^{21} + 21657600k^{20}x + 162792960k^{19}x^2 + 778137600k^{18}x^3 \\
&+ 2645782983k^{17}x^4 + 6789381590k^{16}x^5 + 13626443025k^{15}x^6 \\
&+ 21889330810k^{14}x^7 + 28579049475k^{13}x^8 + 30634381522k^{12}x^9 \\
&+ 27125436630k^{11}x^{10} + 19896883200k^{10}x^{11} + 12088287630k^9x^{12} \\
&+ 6063596590k^8k^{13} + 2494770300k^7x^{14} + 832958400k^6x^{15} \\
&+ 222060150k^5x^{16} + 46134540k^4x^{17} + 7195500k^3x^{18} \\
&+ 792300k^2x^{19} + 54900kx^{20} + 1800x^{21}.
\end{aligned}$$

**Remark 2.4.** *We want to note that Theorem 2.2 and Corollary 2.3 are k-generalizations of Theorem 1 and Remark 2 in [6], respectively.*

**Remark 2.5.** *The left side of inequality (1.14)is better than the left side of inequality (1.19) for $x>1.8157k$ and $k>0$. Also the upper bound in the inequality (1.14) is better than the one in the inequality (1.19) for $0<x<6.58818k$.*

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# Generating Matrix and Sums of Hyperbolic Fibonacci Sequnce

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In this paper, we study the hyperbolic Fibonacci sequence and developed generating matrices for its. First we proved two results on the even sum of the hyperbolic Fibonacci sequence, using the generating matrix approach. We then deduce the odd sum, some identity and recursive formulas for this sequence.

Keywords: Matrix, Hyperbolic number, Hyperbolic Fibonacci number.
2020 Mathematics Subject Classification: 11B37, 11B39, 11B83.

## 1 INTRODUCTION

There has been enormous interest in the research of number sequences due to their abundance and rich properties which had contributed to further research and many applications in number theory, science and nature. (See for example [4], [5], [6], [9], [10], [11], [12], [15], [18], [19]).

The set of hyperbolic numbers $\mathbb{H}$can be described as

$$\mathbb{H} = \left\{ z = x + hy : h \notin \mathbb{R}, h^2 = 1, x, y \in \mathbb{R} \right\}$$

Addition, substruction and multiplication of any two hyperbolic numbers $z_1$ and $z_2$ are defined by

$$\begin{aligned} z_1 \pm z_2 &= (x_1 + hy_1) \pm (x_2 + hy_2) = (x_1 \pm x_2) + h(y_1 \pm y_2) \\ z_1 \times z_2 &= (x_1 + hy_1) \times (x_2 + hy_2) = x_1x_2 + y_1y_2 + h(x_1y_2 + y_1x_2) \end{aligned}$$

and the division of two hyperbolic numbers are given by

$$\frac{z_1}{z_2} = \frac{x_1 + hy_1}{x_2 + hy_2} = \frac{(x_1 + hy_1)(x_2 - hy_2)}{(x_2 + hy_2)(x_2 - hy_2)} = \frac{x_1x_2 + y_1y_2}{x_2^2 - y_2^2} + h\frac{(x_1y_2 + y_1x_2)}{x_2^2 - y_2^2}$$

The hyperbolic conjugation of $z = x + hy$ is defined by

$$\bar{z} = x - hy$$

For more informatin on hyperbolic numbers, see for example ([1], [2], [3], [7], [8], [13], [14], [16], [17])

In [2], author, the hyperbolic Fibonacci sequence defined by

$$\tilde{F}_n = F_n + hF_{n+1}, \quad h^2 = 1$$

with $F_0 = h, F_1 = 1 + h$. That is, The hyperbolic Fibonacci sequence $F_n$ is

$$h, 1 + h, 1 + 2h, 2 + 3h, 3 + 5h, \cdots, (1 + h)F_n + hF_{n-1}, \cdots$$

Consider the generalized hyperbolic Fibonacci sequence $\left\{\tilde{G}_n\right\}$ defined by the recurrence relation

$$\tilde{G}_{n+1} = a\tilde{G}_n + \tilde{G}_{n-1}$$

for $n \geq 1$ where $\tilde{G}_0 = h, \tilde{G}_1 = 1 + h$, and $a$ is any integer.

For the case $a = 1, \left\{\tilde{G}_n\right\} = \left\{\tilde{F}_n\right\}$ where $\tilde{F}_n$ is the $n^{th}$ hyperbolic Fibonacci number.

## 2 Matrix of Hyperbolic Fibonacci Sequence

Given a number sequnce $\{u_n\}$, we call $M$ a generating matrix for the sequnce if for any positive integer $m$, the entries in $M^m$ are numbers or combination of numbers from the sequnce $\{u_n\}$ itself.

The matrix of the hyperbolic Fibonacci sequence is defined by

$$\begin{pmatrix} \tilde{F}_{n+1} & \tilde{F}_n \\ \tilde{F}_n & \tilde{F}_{n-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} \tilde{F}_1 & \tilde{F}_0 \\ \tilde{F}_0 & \tilde{F}_{-1} \end{pmatrix}$$

In this paper, we investigate the mathematical properties of the generalized Hyperbolic Fibonacci sequence $\left\{\tilde{G}_n\right\} = \left\{\tilde{F}_n\right\}$ using generating matrix of form $\begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}$.

Also we obtained formula for even

$$\sum_{i=0}^{n} \tilde{F}_{2i}$$

the odd sum

$$\sum_{i=0}^{n} \tilde{F}_{2i+1}$$

is deduced accordingly.

**Theorem 1** *The sum of the hyperbolic Fibonacci numbers can be expressed as*

$$\sum_{i=0}^{n} \tilde{F}_i = \tilde{F}_{n+2} - (1 + h)$$

**Proof.** By induction methods, for integer $n=1$, the equeation is true. Indeed;

$$\sum_{i=0}^{1}\tilde{F}_i=\tilde{F}_0+\tilde{F}_1=h+(1+h)=1+2h=\tilde{F}_3-(1+h)=(2+3h)-(1+h)$$

Now let the equation be true for $n=k$. That is,

$$\sum_{i=0}^{k}\tilde{F}_i=\tilde{F}_{k+2}-(1+h)$$

In this case, let's show that the equation is also true for $n=k+1$.

$$\begin{aligned}\sum_{i=0}^{k+1}\tilde{F}_i &= \underbrace{\tilde{F}_0+\tilde{F}_1+\tilde{F}_2+\cdots+\tilde{F}_k}_{\tilde{F}_{k+2}-(1+h)}+\tilde{F}_{k+1}\\ &= \tilde{F}_{k+2}-(1+h)+\tilde{F}_{k+1}\\ &= \tilde{F}_{k+3}-(1+h)\end{aligned}$$

This completes the proof of the theorem.

**Lemma 2** *For integer $a$, let*

$$\tilde{G}_{n+1}=a\tilde{G}_n+\tilde{G}_{n-1}$$

*for $n\geq 1$ where $\tilde{G}_0=h,\tilde{G}_1=1+h$. Then for any positive integer $t\geq 2$,*

$$\tilde{G}_{t+2}=\left(a^2+2\right)\tilde{G}_t+\tilde{G}_{t-2}$$

**Proof.**

$$\begin{aligned}\tilde{G}_{t+2} &= a\tilde{G}_{t+1}+\tilde{G}_t\\ &= a\tilde{G}_{t+1}+\tilde{G}_t+\tilde{G}_{t-2}-\tilde{G}_{t-2}\\ &= a\left(a\tilde{G}_t+\tilde{G}_{t-1}\right)+\tilde{G}_t+\left(\tilde{G}_t-a\tilde{G}_{t-1}\right)-\tilde{G}_{t-2}\\ &= \left(a^2+2\right)\tilde{G}_t+\tilde{G}_{t-2}\end{aligned}$$

**Lemma 3** *For any positive integer $n$,*

$$\begin{pmatrix}\tilde{F}_{2n+2} & -\tilde{F}_{2n}\\ \tilde{F}_{2n} & -\tilde{F}_{2n-2}\end{pmatrix}=\begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix}^n\begin{pmatrix}\tilde{F}_2 & -\tilde{F}_0\\ \tilde{F}_0 & -\tilde{F}_{-2}\end{pmatrix}$$

**Proof.** First note that the lemma is true for $n=1$ since

$$\begin{pmatrix}\tilde{F}_4 & -\tilde{F}_2\\ \tilde{F}_2 & -\tilde{F}_0\end{pmatrix}=\begin{pmatrix}3 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\tilde{F}_2 & -\tilde{F}_0\\ \tilde{F}_0 & -\tilde{F}_2\end{pmatrix}$$

Consequently,

$$\begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}^{k+1} \begin{pmatrix} \tilde{F}_2 & -\tilde{F}_0 \\ \tilde{F}_0 & -\tilde{F}_{-2} \end{pmatrix} = \begin{pmatrix} \tilde{F}_{2k+2} & -\tilde{F}_{2k} \\ \tilde{F}_{2k} & -\tilde{F}_{2k-2} \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3\tilde{F}_{2k+2} - \tilde{F}_{2k} & -\tilde{F}_{2k+2} \\ 3\tilde{F}_{2k} - \tilde{F}_{2k-2} & -\tilde{F}_{2k} \end{pmatrix}$$

By Lemma 1 with $a = 1$ and $t = 2k + 2$, we have

$$\tilde{F}_{2k+4} = 3\tilde{F}_{2k+2} - \tilde{F}_{2k}$$

and

$$\tilde{F}_{2k+2} = 3\tilde{F}_{2k} - \tilde{F}_{2k-2}$$

This completes the proof.

**Lemma 4** *Let*

$$E_m = \sum_{i=0}^{m} \widetilde{F}_{2i}$$

*Then for any positive integer $n$,*

$$\begin{pmatrix} 1 & 0 & 0 \\ E_n & \tilde{F}_{2n+2} & -\tilde{F}_{2n} \\ E_{n-1} & \tilde{F}_{2n} & -\tilde{F}_{2n-2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix}^n \begin{pmatrix} 1 & 0 & 0 \\ \tilde{F}_0 & \tilde{F}_2 & -\tilde{F}_0 \\ 0 & \tilde{F}_0 & -\tilde{F}_{-2} \end{pmatrix}$$

**Proof.** First note that the lemma is true for $n = 1$ since

$$\begin{pmatrix} 1 & 0 & 0 \\ E_1 & \tilde{F}_4 & -\tilde{F}_2 \\ E_0 & \tilde{F}_2 & -\tilde{F}_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \tilde{F}_0 & \tilde{F}_2 & -\tilde{F}_0 \\ 0 & \tilde{F}_0 & -\tilde{F}_{-2} \end{pmatrix}$$

Assume the lemma is true for $n = k \geq 2$. Then

$$\begin{pmatrix} 1 & 0 & 0 \\ E_k & \tilde{F}_{2k+2} & -\tilde{F}_{2k} \\ E_{k-1} & \tilde{F}_{2k} & -\tilde{F}_{2k-2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix}^k \begin{pmatrix} 1 & 0 & 0 \\ \tilde{F}_0 & \tilde{F}_2 & -\tilde{F}_0 \\ 0 & \tilde{F}_0 & -\tilde{F}_{-2} \end{pmatrix}$$

Consequently,

$$\begin{aligned} & \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix}^{k+1} \begin{pmatrix} 1 & 0 & 0 \\ \tilde{F}_0 & \tilde{F}_2 & -\tilde{F}_0 \\ 0 & \tilde{F}_0 & -\tilde{F}_{-2} \end{pmatrix} \\ = & \begin{pmatrix} 1 & 0 & 0 \\ E_k & \tilde{F}_{2k+2} & -\tilde{F}_{2k} \\ E_{k-1} & \tilde{F}_{2k} & -\tilde{F}_{2k-2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix} \\ = & \begin{pmatrix} 1 & 0 & 0 \\ E_k + \tilde{F}_{2k+2} & 3\tilde{F}_{2k+2} - \tilde{F}_{2k} & -\tilde{F}_{2k+2} \\ E_{k-1} + \tilde{F}_{2k} & 3\tilde{F}_{2k} - \tilde{F}_{2k-2} & -\tilde{F}_{2k} \end{pmatrix} \end{aligned}$$

By Lemma 1 with $a = 1$, we have $\tilde{F}_{t+2} = 3\tilde{F}_t - \tilde{F}_{t-2}$ for any $t \geq 2$. This completes the proof.

**Theorem 5** *For any integer positive $n$,*

$$\sum_{i=0}^{n} \tilde{F}_{2i} = \tilde{F}_{2n+2} - \tilde{F}_{2n} - 1$$

**Proof.** By induction methods, for integer $n = 1$, the equeation is true. Indeed;

$$\sum_{i=0}^{1} \tilde{F}_{2i} = \tilde{F}_0 + \tilde{F}_2 = h + (1 + 2h) = 1 + 3h = \tilde{F}_4 - \tilde{F}_2 - 1 = (3 + 5h) - (1 + 2h) - 1$$

Now let the equation be true for $n = k$. That is,

$$\sum_{i=0}^{k} \tilde{F}_{2i} = \tilde{F}_{2k+2} - \tilde{F}_{2k} - 1$$

In this case, let's show that the equation is also true for $n = k + 1$.

$$\begin{aligned}
\sum_{i=0}^{k+1} \tilde{F}_{2i} &= \underbrace{\tilde{F}_{2k+2} - \tilde{F}_{2k} - 1}_{\tilde{F}_0+\tilde{F}_2+\tilde{F}_4+\cdots+\tilde{F}_{2k}} + \tilde{F}_{2k+2} \\
&= \tilde{F}_{2k+2} - \tilde{F}_{2k} - 1 + \tilde{F}_{2k+2} \\
&= \tilde{F}_{2k+2} - \tilde{F}_{2k} - 1 + \left(\tilde{F}_{2k+4} - \tilde{F}_{2k+3}\right) \\
&= \tilde{F}_{2k+4} - \left(\tilde{F}_{2k+3}\right) + \tilde{F}_{2k+2} - \tilde{F}_{2k} - 1 \\
&= \tilde{F}_{2k+4} - \left(\tilde{F}_{2k+2} + \tilde{F}_{2k+1}\right) + \tilde{F}_{2k+2} - \tilde{F}_{2k} - 1 \\
&= \tilde{F}_{2k+4} - \left(\tilde{F}_{2k+1} + \tilde{F}_{2k}\right) - 1 \\
&= \tilde{F}_{2k+4} - \tilde{F}_{2k+2} - 1.
\end{aligned}$$

This completes the proof of the theorem.

**Theorem 6** *For any integer positive $n$,*

$$\sum_{i=0}^{n} \tilde{F}_{2i+1} = \tilde{F}_{2n+3} - \tilde{F}_{2n+1} - h$$

**Proof.** By induction methods, for integer $n = 1$, the equeation is true. Indeed;

$$\begin{aligned}
\sum_{i=0}^{1} \tilde{F}_{2i+1} &= \tilde{F}_1 + \tilde{F}_3 = (1 + h) + (2 + 3h) \\
&= 3 + 4h = \tilde{F}_5 - \tilde{F}_3 - h = (5 + 8h) - (2 + 3h) - h.
\end{aligned}$$

Now let the equation be true for $n = k$. That is,

$$\sum_{i=0}^{k} \tilde{F}_{2i+1} = \tilde{F}_{2k+3} - \tilde{F}_{2k+1} - h$$

In this case, let's show that the equation is also true for $n = k + 1$.

$$\begin{aligned}
\sum_{i=0}^{k+1} \tilde{F}_{2i+1} &= \underbrace{\tilde{F}_3 + \tilde{F}_5 + \cdots + \tilde{F}_{2k+1}}_{\tilde{F}_{2k+3}-\tilde{F}_{2k+1}} + \tilde{F}_{2k+3} \\
&= \tilde{F}_{2k+3} - \tilde{F}_{2k+1} - h + \tilde{F}_{2k+3} \\
&= \tilde{F}_{2k+3} - \tilde{F}_{2k+1} - h + \left(\tilde{F}_{2k+5} - \tilde{F}_{2k+4}\right) \\
&= \tilde{F}_{2k+5} - \left(\tilde{F}_{2k+3} + \tilde{F}_{2k+2}\right) + \tilde{F}_{2k+3} - \tilde{F}_{2k+1} - h \\
&= \tilde{F}_{2k+5} - \left(\tilde{F}_{2k+3} + \tilde{F}_{2k+2}\right) + \tilde{F}_{2k+3} - \tilde{F}_{2k+1} - h \\
&= \tilde{F}_{2k+5} - \left(\tilde{F}_{2k+2} + \tilde{F}_{2k+1}\right) - h \\
&= \tilde{F}_{2k+5} - \tilde{F}_{2k+3} - h.
\end{aligned}$$

This completes the proof of the theorem.

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| | Asst. Prof. Dr. Ayşenur UÇAR<br><br>Doğuş University, Türkiye |
| | Asst. Prof. Dr. Semra YURTTANÇIKMAZ<br><br>Atatürk University, Türkiye |
| | Asst. Prof. Dr. Sibel TURANLI<br><br>Atatürk University, Türkiye |
| Mathematical Analysis | Prof. Dr. Doria AFFANE<br><br>University of Jijel, Algeria |
| | Prof. Dr. Halit ORHAN<br><br>Atatürk University, Türkiye |

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|---|---|
| | Prof. Dr. İsa Yıldırım<br>Atatürk University, Türkiye |
| | Prof. Dr. Mustapha Fateh YAROU<br>University of Jijel, Algeria |
| | Prof. Dr. Sezqin Akbulut<br>Atatürk University, Türkiye |
| | Assoc. Prof. Dr. Asifa TASSADDİQ<br>Majmaah University, Saudi Arabia |
| | Asst. Prof. Dr. Mohammad Esmael SAMEİ<br>Bu-Ali Sina University, Iran |
| Mathematical Modelling | Prof. Dr. George ANASTASSİOU<br>The University of Memphis, USA |
| | Prof. Dr. Jordan HRİSTOV<br>University Of Chemical Tecnology And Metallurgy, Bulgaria |
| | Asst. Prof. Dr. Abdelhakim IDIR<br>Mohamed Boudiaf University of M'sila, Algeria |
| | Asst. Prof. Dr. Chokkalingam RAVİCHANDRAN<br>Kongunadu Arts and Science College, India |
| | Asst. Prof. Dr. Devendra KUMAR<br>University of Rajasthan, India |
| Mathematical Statistics | Asst. Prof. Dr. Mohammad Reza MAHMOUDİ<br>Fasa University, Iran |
| Mathematics Education | Prof. Dr. Alper Cihan KONYALIOĞLU<br>Atatürk University, Türkiye |

| | |
|---|---|
| | Prof. Dr. Alper ÇİLTAŞ<br><br>Atatürk University, Türkiye |
| Quantum Mechanics | Prof. Dr. Jihad ASAD<br><br>Palestine Technical University, Palestine |
| Topology | Prof. Dr. Ceren Sultan ELMALI<br><br>Erzurum Technical University, Türkiye |
| | Prof. Dr. Tamer UĞUR<br><br>Atatürk University, Türkiye |
| | Asst. Prof. Dr. Kadirhan POLAT<br><br>Ağrı İbrahim Çeçen University, Türkiye |
| | Asst. Prof. Dr. Nuray GÜL<br><br>Bitlis Eren University, Türkiye |

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# SCIENTIFIC PROGRAM

*Note that all times listed on the schedule are shown as UTC/GMT +3 , i.e. the time in TURKEY .*
*Please, gauge the time difference to your time zone.*

| May 23, 2022 | |
|---|---|
| **09:30--11:00** | Openning Ceremony |
| **11:00--11:30** | Invited Speaker |
| **11:30--12:00** | Invited Speaker |
| **12:00--13:00** | Break |
| **13:00--13:30** | Invited Speaker |
| **13:45--15:00** | Session 1 |
| **15:15--15:45** | Invited Speaker |
| **16:00--17:15** | Session 2 |
| **May 24, 2022** | |
| **09:30--10:00** | Invited Speaker |
| **10:15--12:00** | Session 1 |
| **12:00--13:00** | Break |
| **13:00--13:30** | Invited Speaker |
| **13:45--15:00** | Session 2 |
| **15:15--16:30** | Session 3 |
| **May 25, 2022** | |
| **09:30--10:00** | Invited Speaker |
| **10:15--12:00** | Session 1 |
| **12:00--13:00** | Break |
| **13:00--13:30** | Invited Speaker |
| **13:45--15:00** | Session 2 |
| **15:15--15:45** | Invited Speaker |
| **16:00--17:15** | Sessions 3 |
| **17:30--18:00** | Clossing Ceremony |

**Note that all times listed on the schedule are shown as UTC/GMT +3, i.e. the time in TURKEY.**

# May 23, 2022

| Link | | | |
|---|---|---|---|
| MAIN HALL | 09:30-11:00 | Openning Ceremony | |
| MAIN HALL | 11:00-11:30 | **Prof. Dr. Ahmet Işık (Kırıkkale University-Türkiye)**<br>**Mathematics and Mathematics Education** | |
| MAIN HALL | 11:30-12:00 | **Prof.Dr. Bismark Singh (Friedrich-Alexander University-Germany)**<br>**Optimization Models for Pandemic Response Planning** | |
| | 12:00-13:00 | Break | |
| | | | |
| Lecture Room | | Invited Speaker | Chairman |
| MAIN HALL | 13:00-13:30 | **Prof. Dr. Arif Salimov (Bakü University – Azerbaijan)**<br>**New Developments in the Theory of Lifts** | Kürşat AKBULUT |
| | 13:30-13:45 | Break | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 1 | SESSION 1 | 13:45-14:00 | Some Density Properties in Bitopological Context<br>**Necati Can ACIKGÖZ** , Ceren Sultan ELMALI | Tamer UĞUR |
| HALL 1 | SESSION 1 | 14:00-14:15 | Fixed-Point Theorems in Extended Fuzzy Metric Spaces Via Some Fuzzy Contractive Mappings<br>**Meryem Şenocak,** Erdal Güner | Tamer UĞUR |
| HALL 1 | SESSION 1 | 14:15-14:30 | The Attitudes and Self-Efficiens of High School Students Continued the Distance Mathematics Course Against Distance Education During the COVID-19 Pandemic<br>**Başak Bor Akbulut** | Tamer UĞUR |
| HALL 1 | SESSION 1 | 14:30-14:45 | Historical-Philosophical Development and Teaching of Mathematical Objects<br>**Fatih Taş** | Tamer UĞUR |
| HALL 1 | SESSION 1 | 14:45-15:00 | On the Controllability of Some Systems on Lie Groups<br>**Okan Duman** | Tamer UĞUR |
| | | 15:00-15:15 | Break | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 1 | 13:45-14:00 | On a solvable system of rational difference equations of higher order<br>**Merve Kara,** Yasin Yazlik | Alper Cihan KONYALIOĞLU |
| | | 14:00-14:15 | Blow up of solution of a nonlinear wave equation with general source and damping terms<br>**Boulmerka Imane** | |
| | | 14:15-14:30 | Global non-existence of solutions for a nonlinear viscoelastic plate equation with a p(x, t)−Laplacian operator in the presence of time delay<br>**Merah Ahlem,** Mesloub Fatiha | |
| | | 14:30-14:45 | Schauder and Banach fixed point theorem for semilinear fractional problem<br>**Chaima Saadi,** Hakim Lakhal, Kamel Slimani | |
| | | 14:45-15:00 | Solving the absolute value equation based on a new smoothing function<br>**Randa Chalekh,** EL Amir Djeffal | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 1 | 13:45-14:00 | Solving One-Dimensional Bratu's Problem via Kashuri Fundo Decomposition Method<br>Haldun Alpaslan Peker, **Fatma Aybike Çuha** | Kürşat AKBULUT |
| | | 14:00-14:15 | On predictors of partial parameters under a partitioned linear model and its reduced models<br>Nesrin Güler, **Melek Eriş Büyükkaya,** Melike Yiğit | |
| | | 14:15-14:30 | Optimal control of a fractional SIR model under the effect of nonlinear incidence and recovery rates<br>Fatma Soytürk, **Derya Avcı** | |
| | | 14:30-14:45 | Existence and uniqueness results for a revisited Nicholson's blowflies model with two different variable delays and a nonlinear harvesting term<br>**Ahleme Bouakkaz,** Rabah Khemis | |
| | | 14:45-15:00 | General decay of solutions in one-dimensional porous-elastic system with memory and distributed delay term with second sound<br>**Fares Yazid,** Fatima Siham Djeradi | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 1 | SESSION 2 | 15:15-15:30 | The Drazin Inverse for Closed Linear Operators<br>**Mohammed Drissi-Alami** ,Mohammed Kachad | Tane VERGİLİ |
| | | 15:30-15:45 | Some Fixed Point Results in Soft Fuzzy Metric Spaces<br>**Merve İnce** , Ferhan Şola Erduran | |
| | | 15:45-16:00 | Some Fixed Point Theorems on O-Complete Metric Spaces<br>**Kübra ÖZKAN** | |
| | | 16:00-16:15 | Homotopy and Descriptive Homotopy in Computational Proximity<br>**Tane VERGİLİ,** James Francis Peters | |
| | | 16:15-16:30 | Trellis theory and some new results<br>**Abdelkrim Mehenni** , Lemnaouar Zedam | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 2 | 15:15-15:30 | Existence and nonexistence of positive solutions to a fractional parabolic problem with singular weight at the boundary<br>**Kheireddine Biroud** | Mesut KARABACAK |
| | | 15:30-15:45 | On the boundary observability and controllability of the wave equation in some non-cylindrical domains<br>**Seyf Eddine Ghenimi, Abdelmouhcene Sengouga** | |
| | | 15:45-16:00 | Existence and uniqueness results for nonlinear hybrid implicit Caputo-Hadamard fractional differential equations<br>**Chahra Kechar, Abdelouaheb Ardjouni** | |
| | | 16:00-16:15 | Complexity analysis of a primal-dual interior-point method for convex quadratic optimization based on a new hyperbolic<br>**Youssra Bouhenache,** Wided Chikouche, Imene Toui | |
| | | 16:15-16:30 | On the asymptotic behaviour of a non-local eigenvalue problem<br>**Ahlem Yahiaoui, Senoussi Guesmia, Abdelmouhcene Sengouga** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 2 | 15:15-15:30 | Growth of solutions of linear fractional differential equations with polynomial coefficients<br>**Saada Hamouda,** Sofiane Mahmoudi | İsa YILDIRIM |
| | | 15:30-15:45 | Non autonomous iterative differential inclusion<br>**Ghalia Samia,** Doria Affane | |
| | | 15:45-16:00 | Semi continuous perturbations for nonconvex sweeping process<br>**Hanane Chouial,** Mustapha Fateh Yarou | |
| | | 16:00-16:15 | Non-convex valued perturbation of first order problems with maximal monotone operators<br>**Fatima Fennour,** Soumia Saidi | |
| | | 16:15-16:30 | On unique solvability and Picard's iterative method for absolute value equations<br>**Nassima Anane,** Mohamed Achache | |

| Lecture Room | | Invited Speaker | Chairman |
|---|---|---|---|
| MAIN HALL | 16:45-17:15 | **Prof. Dr. James Peters (University of Manitoba - Canada)**<br>**Geometric Realization of Path Cycles as Flow Cycles** | Tane VERGİLİ |

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# May 24, 2022

| Lecture Room | Session | Invited Speaker | Chairman |
|---|---|---|---|
| MAIN HALL | 9:30-10:00 | **Prof. Dr. Bayram Şahin (Ege University – Türkiye)**<br>**Conformal Riemannian Maps from Kaehler Manifolds to Riemannian Manifolds** | Engin ÖZKAN |
| | 10:00-10:15 | Break | |

| Lecture Room | Session | | Speakers | Chairman |
|---|---|---|---|---|
| HALL 1 | SESSION 1 | 10:15-10:30 | Some Aspects of Interchanging Difference Equation Orders<br>**Anthony G. Shannon , Engin Ozkan** | Engin ÖZKAN |
| | | 10:30-10:45 | On Quasi Ideals of Nearness Semigroups<br>**Ozlem TEKİN** | |
| | | 10:45-11:00 | Generalized Fibonacci Polynomials Associated with Finite Operators<br>**Emrah POLATLI** | |
| | | 11:00-11:15 | A New Generalization Of The Min and Max Matrices<br>**Nazlıhan TERZİOĞLU** , Can KIZILATEŞ | |
| | | 11:15-11:30 | Number of Spanning Trees in Generalized Farey Graphs<br>**Ibrahim GOKCAN** | |
| | | 11:30-11:45 | Simultaneously Square and Centered Square Numbers Related with Pell and Lucas Numbers<br>**Ahmet EMİN** | |
| | | 11:45-12:00 | Centered Polygonal Numbers and Polygonal Numbers<br>**Umit Sarp** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 1 | 10:15-10:30 | On Eight Solvable Systems of Difference Equations in Terms of Generalized Padovan Sequences<br>**Merve Kara, Yasin Yazlik** | Alper ÇİLTAŞ |
| | | 10:30-10:45 | A pointwise Carleman inequality for the general ultrahyperbolic Schrödinger equation<br>**Özlem Kaytmaz** | |
| | | 10:45-11:00 | Solvability of an Inverse Problem for a Kinetic Equation on a Riemannian Manifold<br>**Ismet Gölgeleyen** | |
| | | 11:00-11:15 | Numerical Solution of simple mechanical systems with deep learning<br>**Tayfun Ünal,** Ayten Irem Işık, Ünver Çiftçi | |
| | | 11:15-11:30 | Uniqueness of solution of an inverse problem for the ultrahyperbolic Schrödinger equation<br>**Özlem Kaytmaz** | |
| | | 11:30-11:45 | A finite difference method based on the operator for the numerical solution of an inverse source problem backward in time<br>**Ali Ugur Sazaklioglu** | |
| | | 11:45-12:00 | Feedback stabilization of bilinear systems<br>**Ayoub Cheddour** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 1 | 10:15-10:30 | Simpson Type Inequalities for Katugampola Fractional Integral<br>**Zeynep Şanlı** | Vehpi Yıldırım |
| | | 10:30-10:45 | Some completely monotonicity properties and related inequalities involving k-trigamma and k-tetragamma functions<br>**Emrah Yıldırım** | |
| | | 10:45-11:00 | Existence result of a Capacity Solution for a Nonlinear Parabolic-Elliptic System<br>**İbrahim Dahi,** Moulay Rchid Sidi Ammi | |
| | | 11:00-11:15 | Jacobi Last Multiplier Method for Optimal Growth model with the Environmental Asset<br>**Gülden GÜN POLAT** | |
| | | 11:15-11:30 | Investigating the Impact of Bariatric Surgery on Lipid and Glucose Absorption via Mathematical Modeling<br>**Sedanur Köksal,** Vehpi Yıldırım | |
| | | 11:30-11:45 | Mathematical Model of COVID-19 with Imperfect Vaccine and Virus Mutation<br>**Ceren Gürbüz**, Sebaheddin Sevgin | |
| | | 11:45-12:00 | Analyzing Neimark-Sacker Bifurcation and Stability for a Discrete-Time Prey- Predator Model with Allee Effect<br>**Nihal Öztürk,** Figen Kangalgil, Nilüfer Topsakal | |

| Link | | | |
|---|---|---|---|
| Lecture Room | | Invited Speaker | Chairman |
| MAIN HALL | 13:00-13:30 | **Prof. Dr. Boaz Tsaban (Bar Ilan University - Israel )**<br>**Selection Principles in Mathematics** | Nejmi CENGİZ |
| | 13:30-13:45 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 1 | SESSION 2 | 13:45-14:00 | Universal Covering of a Lie Group<br>**Merve Ersoy,** Eyüp Kızıl | Nejmi CENGİZ |
| | | 14:00-14:15 | Inquiry-Based Learning: A Bibliometric Analysis<br>**Seher Aslancı** | |
| | | 14:15-14:30 | Examining the perceptions of anatolian vocational high school students on mathematics through metaphors<br>**Ömer Demirci, Özlem Demirci** | |
| | | 14:30-14:45 | Associated curves of a framed curve in Euclidean 3-space<br>**Zeynep Bülbül,** Mustafa Düldül | |
| | | 14:45-15:00 | On the cosine curve as 4th and 6th order BÈzier curve in E2□<br>**Şeyda Kılıçoğlu** | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 2 | 13:45-14:00 | Rational Decay Rate for the Wave equation with non-neglected Density<br>**Karima LAOUBI** | Şeyda KILIÇOĞLU |
| | | 14:00-14:15 | A Dynamic electroviscoelastic problem with thermal effects<br>**Sihem Smata, Nemira Lebri** | |
| | | 14:15-14:30 | Limit cycles of a class of planar polynomial differential systems<br>**Amel Boulfoul,** Nassima Debz, Abdelhak Berkane | |
| | | 14:30-14:45 | A Derivative-Free algorithm for continuous global optimization<br>**Raouf Ziadi** | |
| | | 14:45-15:00 | Well-posedness and energy decay of swelling porous elastic soilswith a second sound and distributed delay term<br>**Sabah Baibeche** | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 2 | 13:45-14:00 | A characterization of open distance pattern uniform chordal graphs and distance hereditary graphs<br>**Bibin K. Jose** | Esra ALTINTAŞ |
| | | 14:00-14:15 | Approximate controllability results for Caputo fractional Volterra-Fredholm integro-differential systems of order $1 < r < 2$<br>**M. Mohan Raja,** V. Vijayakumar | |
| | | 14:15-14:30 | Uniform well-posedness and stability for fractional Navier-Stokes equations with Coriolis force in critical Fourier-Besov-Morrey spaces<br>**Ahmed El Idrissi,** Brahim El Boukari, Jalila El Ghordaf | |
| | | 14:30-14:45 | A new primal-dual interior-point algorithm for linear programming<br>**L. Derbal,** Z. Kebbiche | |
| | | 14:45-15:00 | Some new results on periodic solutions for a periodic delay hematopoiesis model with a unimodal production function<br>**Rabah Khemis,** Ahleme Bouakkaz | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| **Lecture Room** | **Session** | | **Speakers** | **Chairman** |
| HALL 1 | SESSION 3 | 15:15-15:30 | Relative cohomology spaces for some osp(n\|2)-modules.□<br>**Wafa Mtaouaa,** Didier Arnal, Mabrouk Ben Ammar, Zeineb Selmi 〖 〗 | Abdullah ÇAĞMAN |
| | | 15:30-15:45 | On integral bases and monogeneity of certain pure number fields defined by x^p ^r-a<br>**Omar Kchit** , Hanan Choulli , Lhoussain El Fadil | |
| | | 15:45-16:00 | Groups whose proper subgroups of infinite rank are hypercentral-by-finite<br>**Amel Dilmi** , Nadir Trabelsi | |
| | | 16:00-16:15 | Diophantine approximation by prime numbers of a special form<br>**Tatiana Todorova** | |
| | | 16:15-16:30 | Symmetric functions for (p, q)-numbers and Pell Lucas polynomials<br>**Meryem Bouzeraib** , Ali Boussayoud | |

| Link | | | | |
|---|---|---|---|---|
| **Lecture Room** | **Session** | | **Speakers** | **Chairman** |
| HALL 2 | SESSION 3 | 15:15-15:30 | Stabilization for wave beam equation with a local Degenerated Kelvin-Voigt Damping<br>**Rania Yahia** | S.Şule ŞENER |
| | | 15:30-15:45 | Growth of solutions of linear fractional differential equations with entire coefficients<br>**Sofiane Mahmoudi, Saada Hamouda** | |
| | | 15:45-16:00 | Sufficient conditions for global asymptotic stability of a kind of nonlinear neutral differential equations<br>**Benhadri Mimia** | |
| | | 16:00-16:15 | A Solution Algorithm for An Inverse Problem for the Kinetic Equation which Involves Poisson Bracket<br>**Muhammed Hasdemir,** İsmet Gölgeleyen | |
| | | 16:15-16:30 | Dynamical behavior of a differential-algebraic system with fractional order<br>**Kerioui Nadjah** | |

| Link | | | | |
|---|---|---|---|---|
| **Lecture Room** | **Session** | | **Speakers** | **Chairman** |
| HALL 3 | SESSION 3 | 15:15-15:30 | Cohen positive strongly p-summing m-homogeneous polynomials from a tensor viewpoint<br>Halima Hamdi, **Amar Belacel** | Nilüfer TOPSAKAL |
| | | 15:30-15:45 | Truncated condition for second order perturbed sweeping process<br>**Imene Mecemma,** Sabrina Lounis, Mostapha Fateh Yarou | |
| | | 15:45-16:00 | An existence result for a class of nonconvex second order differential inclusions **Nora Fetouci,** M. F. Yarou | |
| | | 16:00-16:15 | Existence problem for first order evolution inclusion<br>**Nouha Boudjerida,** Doria Affane, Yarou Mustapha Fateh | |
| | | 16:15-16:30 | Some fixed point theorems for a Generalized cyclic (α,f,ϕ,ψ)−contractive mapping in b−Metric-Like Spaces<br>**Merad Souheib** | |

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# May 25, 2022

| Link | | | | |
|---|---|---|---|---|
| **Lecture Room** | **Session** | | **Invited Speaker** | **Chairman** |
| MAIN HALL | | 9:30-10:00 | **Assoc. Prof. Dr. Murat KİRİŞÇİ (Istanbul University - Cerrahpaşa - Türkiye)**<br>**On the artificial intelligence, big data, blockchain technologies in medicine** | Sait TAŞ |
| | | 10:00-10:15 | **Break** | |

| Lecture Room | | Session | Speakers | Chairman |
|---|---|---|---|---|
| HALL 1 | SESSION 1 | 10:15-10:30 | A New Approach Tubular Surface with a new frame in G3<br>**Gökhan Mumcu,** Ali Çakmak, Tülay Erişir, Sezai Kızıltuğ | Sait TAŞ |
| | | 10:30-10:45 | Receiving Student Opinions Within The Scope of Geometry Lessons Taught Using Activities Regarding Different Demo<br>**Esra Altıntaş,** Şükrü İlgün, Sümeyye Güneş | |
| | | 10:45-11:00 | Examination of Preservice Teachers' Mathematical Thinking and Modeling Skills<br>**Zeynep İĞDELİ,** Okan KUZU, Osman ÇİL | |
| | | 11:00-11:15 | Bibliometric Analysis of Scientific Studies on "Noticing Skill" in Mathematics Education<br>**Ercan Dede,** Ercan Özdemir | |
| | | 11:15-11:30 | Examination of Mathematics Questions in Secondary Education Transition Exam According to Revised Bloom Taxonomy and Middle School Mathematics Curriculum Objectives<br>**Zeynep Büşra Üzümcü,** Ali Sabri İpek | |
| | | 11:30-11:45 | Reflections of Developed Problem Posing Based Active Learning Activities in the Teaching Process: Example of Fractions<br>**Hatice Polat,** Merve Özkaya | |
| | | 11:45-12:00 | Examining Secondary School 7th Grade Mathematics Activities within the Scope of Harezmian Education Model and Obtaining Students' Opinions<br>**Esra Altıntaş, Şükrü İlgün, Sümeyye Güneş** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | | Session | Speakers | Chairman |
| HALL 2 | SESSION 1 | 10:15-10:30 | Comparison of Two Effective Methods on Numerical Solutions of Differential Equations<br>**Özlem Soylu, Onur Karaoğlu** | Nuray GÜL |
| | | 10:30-10:45 | The Dirichlet Problem for the Polyanalytic Equations in a Ring Domain<br>**İlker Gençtürk** | |
| | | 10:45-11:00 | A Finite Difference Scheme for Singularly Perturbed Neutral Type Differential Equations<br>**Yilmaz Ekinci,** Erkan Cimen, Musa Cakir | |
| | | 11:00-11:15 | A Numerical Approach for System of Ordinary Differential Equations<br>**Şevket Üncü,** Erkan Cimen | |
| | | 11:15-11:30 | Solving Abel's Integral Equation by Kashuri Fundo Transform<br>**Fatma Aybike Çuha,** Haldun Alpaslan Peker | |
| | | 11:30-11:45 | Immigration and Qualitative Behavior of a Two-Dimensional Discrete-Time Model<br>**Seval Işık**, Figen Kangalgil, Feda Gümüşboğa | |
| | | 11:45-12:00 | B-spline method for solving fractional delay differential equations<br>**Mwaffag Sharadga,** Muhammed Syam, Ishak Hashim | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | | Session | Speakers | Chairman |
| HALL 3 | SESSION 1 | 10:15-10:30 | Generalized Spherical Fuzzy Hamacher Aggregation Operators<br>**Elif Güner,** Halis Aygün | Engin ÖZKAN |
| | | 10:30-10:45 | Fekete-Szegö problem for a subclass of bi-univalent functions associated with Gegenbauer polynomials<br>Murat Çağlar, **Mucahit Buyankara** | |
| | | 10:45-11:00 | A numerical approach for a class of singularly perturbed differential-difference equation<br>**Erkan Cimen** | |
| | | 11:00-11:15 | Durrmeyer-type generalization of some linear positive operators<br>Kadir Kanat, Melek Sofyalıoğlu, **Selin Erdal** | |
| | | 11:15-11:30 | Local existence of solutions for a quasilinear hyperbolic equation involving the p-laplacian operator<br>**Abir Bounaama** | |
| | | 11:30-11:45 | A generalized exponential expansion method to simulate two third-order KdV-type equations<br>**Riadh Hedli,** Fella Berrimi | |
| | | 11:45-12:00 | Comparatıve numerical study between line search methods and minorant functions ın barrier logarithmic methods for linear programming<br>**Assma LEULMI,** Soumya LEULMI | |

| Link | | | |
|---|---|---|---|
| Lecture Room | | Invited Speaker | Chairman |
| MAIN HALL | 13:00-13:30 | **Prof. Dr. Josef Mikesh (Palacky University - Czech Republic)**<br>**Geodesics Mappings and Their Generalizations** | Yaprak DERİCİOĞLU |
| | 13:30-13:45 | **Break** | |

| Link | | | | |
|---|---|---|---|---|
| Lecture Room | Session | | Speakers | Chairman |
| HALL 1 | SESSION 2 | 13:45-14:00 | Generalized Fermi Derivative on Surfaces in Euclidean 3-Spaces<br>**Ayşenur Uçar,** Fatma Karakuş | Yaprak DERİCİOĞLU |
| | | 14:00-14:15 | Generalized Fermi Derivative with Regard to Hypersurfaces<br>**Ayşenur Uçar,** Fatma Karakuş | |
| | | 14:15-14:30 | The Study of Pre- Service Mathematics teachers Approach on Students Misunderstandings for Four Possible Answers to Solve a Problem<br>**Samad Shabanifar,** Manouchehr Behboudi Asl | |
| | | 14:30-14:45 | Parallel Transported Along Dual Lorentzian Spacelike And Timelike Curves<br>**Fatma Karakuş,** Tevfik Şahin, Yusuf Yaylı | |
| | | 14:45-15:00 | Darboux Frame with Respect to Generalized Fermi-Walker Derivative<br>**Ayşenur Uçar,** Fatma Karakuş, Yusuf Yaylı | |
| | | 15:00-15:15 | **Break** | |

| Link | | | | |
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| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 2 | 13:45-14:00 | Analysis of a Electro-Elastic contact problem with wear and unilateral constraint<br>**Laldja Benziane, Nemira Lebri** | Nazlı KARACA |
| | | 14:00-14:15 | Berge equilibrium in random bi-matrix game<br>**Djebara Sabiha, Achemine Farida, Zerdani Ouiza** | |
| | | 14:15-14:30 | Resolution a problem of quantum mechanics in fractional dimensional space<br>**Hadjer Merad,** Mihamed Hadj Moussa | |
| | | 14:30-14:45 | Existence and uniqueness of positive periodic solutions for a kind of first order neutral functional differential equations with variable delays<br>**Lynda Mezghiche,** Rabah Khemis, Ahleme Bouakkaz | |
| | | 14:45-15:00 | Unlike classical processes, linearizing first, then discretizing is the better process to solve systems of nonlinear integral equations<br>**I. Sedka,** A. Khellaf, M. Z. Aissaoui | |
| | | 15:00-15:15 | **Break** | |

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| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 2 | 13:45-14:00 | Existence, uniqueness and stability results for a neutral Mackey-Glass type delay differential equation with an iterative production term<br>**Marwa Khemis,** Ahleme Bouakkaz | Seher ASLANCI |
| | | 14:00-14:15 | Local linear estimation of a conditional quantile for randomly censored functional dependent data<br>**Sarra Leulmi,** Farid Leulmi | |
| | | 14:15-14:30 | Modelling of Pancreatic Beta-cells with Gap-junction<br>**Murat An,** Vehbi Yıldırım | |
| | | 14:30-14:45 | Existence, uniqueness and stability of solutions for a first order iterative functional differential equation<br>**Safa Chouaf,** Rabah Khemis, Ahleme Bouakkaz | |
| | | 14:45-15:00 | An Existence Study for a Tripled System with p-Laplacian Involving φ-Caputo Derivatives<br>**Hamid Beddani** | |
| | | 15:00-15:15 | **Break** | |

| Link | | | |
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| Lecture Room | | Invited Speaker | Chairman |
| HALL 1 | 15:15-15:45 | **Prof. Dr. Ljubisa D. R.Kocinac (University of Nis- Serbia)**<br>**More on Set Versions of Star Selection Principles** | Ayşenur UÇAR |
| | 15:45-16:00 | Break | |

| Link | | | | |
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| Lecture Room | Session | | Speakers | Chairman |
| HALL 1 | SESSION 3 | 16:00-16:15 | Non-identical Chaotic Fractional Order Systems : Adaptive Feedback Control and Synchronization<br>**Hamri Douaa,** Hannachi Fareh | Ayşenur UÇAR |
| | | 16:15-16:30 | Recursive Double Kernel Estimator of the Conditional Quantile for Functional Ergodic Data<br>**Imane Bouazza,** Fatima Benziadi, Toufik Guendouzi | |
| | | 16:30-16:45 | An approach to the Diophantine equations with integer sequences<br>**Abdullah Çağman** | |
| | | 16:45-17:00 | A note on L and R topologies<br>**Kadirhan POLAT** | |
| | | 17:00-17:15 | An examination of the conceptual knowledge of teacher candidates in the elementary mathematics program regarding the concept of ratio<br>**Berna Yıldızhan,** Erhan Ertekin | |

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| Lecture Room | Session | | Speakers | Chairman |
| HALL 2 | SESSION 3 | 16:00-16:15 | Under Truncated Random Data, Nonparametric Relative Error Estimation Via Functional Regressor Using The k Nearest Neighbors Smoothing<br>**Nadjet Bellatrach,** Wahiba Bouabsa | Fatma SAĞSÖZ |
| | | 16:15-16:30 | Differential equations of divergence form by Topological Degree in Musielak-Orlicz-Sobolev Spaces<br>**Mustapha Ait Hammou** | |
| | | 16:30-16:45 | Analysis and applications of the proportional Caputo derivative and integral<br>**Abdellatif Boutiara** | |
| | | 16.45-17:00 | Limiting spectral distribution of large dimensional random matrices of linear processes<br>**Zahira Khettab** | |
| | | 17:00-17:15 | Differential Entropy of Continuous Distributions<br>**Ahmet Demiralp,** Yunus Bulut | |
| | | 17:15-17:30 | On deformed lifts<br>**Seher Aslancı** | |

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| Lecture Room | Session | | Speakers | Chairman |
| HALL 3 | SESSION 3 | 16:00-16:15 | On the relationship between the degree of coefficients and the growth of solutions of ultrametric q-difference equations<br>**Boughaba Houda,** Zerzaihi Tahar | Semra KAYA YURTTANÇIKMAZ |
| | | 16:15-16:30 | A class of differential inclusions involving an antimonotone condition<br>**Soumia Saidi** | |
| | | 16:30-16:45 | Existence and multiplicity result for general Steklov problem<br>**Mariya Sadiki,** Belhadj Karim | |
| | | 16:45-17:00 | Liouville theorems for stable at infinity solutions of Lane-Emden system<br>**Foued Mtiri,** Dong Ye | |
| | | 17:00-17:15 | Locally I- connectedness<br>**Selahattin Kılınç** | |
| | | 17:15-17:30 | Generating matrix and sum of Hyperbolic Fibonacci sequences<br>**Sait Taş** | |